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MINING MULTI-LAYERED NETWORKS

APPLICATIONS TO THE PORTUGUESE URBAN SYSTEM AND EU DOMAINS

by

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Master Thesis in Data Analytics – Modelling, Data Analysis and Decision Support Systems

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"Para ser grande, sê inteiro: nada
Teu exagera ou exclui.

Sê todo em cada coisa. Põe quanto és
No mínimo que fazes.

Assim em cada lago a lua toda
Brilha, porque alta vive."
Fernando Pessoa

João Marcelo Fernandes Costa was born in Fafe, Portugal, on November 11th, 1991. He has taken his higher education pathway in the University of Porto with a degree in Economics by the School of Economics and Management (2012). In February 2016, he finished the curricular course in Studies in Modelling, Data Analysis and Decision Support Systems in the same university, which made him a Master candidate.

Currently, João Costa is employed by the *European Central Bank* (ECB) at the Analytical Domains Applications Division, Directorate General-Information Systems. In the past, he was an accountant assistant at *PricewatterhouseCoopers* and he collaborated with *Caixa Geral de Depósitos* as administrative at the private banking commercial area. During his academic career, some of the most relevant developed projects were the publication of the encyclopaedic book "*Os 27 Países da União Europeia*" (2009) and had worked in a data analysis paper *"Healthcare Systems in Developed Countries"* (2015). Furthermore, he had two main international academic experiences: the participation in the Comenius Project "My Job in the European Space" in Sandviken, Sweden (2009), under the graduation of secondary education in High School of Fafe and the realization of Erasmus Mobility Program in the Academia de Studii Economice din Bucureşti – Bucharest, Romania (2011/12).

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"Aos que a felicidade
É sol, virá a noite.
Mas ao que nada espera
Tudo que vem é grato."
Fernando Pessoa

ABSTRACT

The main purpose of this thesis is to emphasise the importance of a multi-layered approach to a network analysis, by applying the most appropriate metrics to two case studies: a territorial network analysis capturing several insights of the Portuguese society and a study identifying the most important and influential leading countries of the European Union.

On the one hand, by giving the basis for monolayer and multilayer network analysis, the researcher develops a solid selection of the main concepts and analytical measures, bearing in mind the general structure of different types of networks. On the other hand, these frameworks are tested in two real multilayer networks, showing the validity and the meaningfulness of the introduced measures, as important and non-random information to mine complex phenomena.

This paper examines all the 308 Portuguese municipalities in some economic features that describe a society (work and education) and the current EU-28 countries composition in important priorities of union policies such as commercial trade, foreign direct investment and migration and remittance flows.

For both national and European schemes, after the analysis of each variable, where communities are detected individually, an adequate method is implemented to detect communities under the perspective of a multilayer network. Subsequently, the multilayer outcomes are compared with the monolayer ones. The application of a multi-layered network allows inferring spatial patterns in a way more consistent in comparison to the geographical structure deduced from each network layer taken separately. Thus, the research problem underlying this dissertation contributes to a structured analysis model to identify the appropriate analytical measures to derive the Portuguese urban system and the European Union global structures.

KEYWORDS: Network Analysis, Mining Multi-layered Networks, Community Detection, Territorial Network

ABBREVIATIONS

A glossary with business and technical terms referenced in this document is available below.

ACRONYM	DETAIL		
SNA	Social Network Analysis		
INSNA	International Network for Social Network Analysis		
WITS	World Integrated Trade Solution		
UNCTAD	United Nations Conference on Trade and Development		
ITC	International Trade Centre		
UNSD	United Nations Statistics Division		
WTO	World Trade Organization		
CESAP	Carta de Equipamentos e Serviços de Apoio à População		
NUTS	Nomenclature of Territorial Units for Statistics		
BPM5	Balance of Payments Manual: Fifth Edition (1993)		
OECD	Organisation for Economic Co-operation and Development		
EU	European Union		
AUT	Austria	**ITA**	Italy
BEL	Belgium	**LVA**	Latvia
BGR	Bulgaria	**LTU**	Lithuania
HRV	Croatia	**LUX**	Luxembourg
CYP	Cyprus	**MLT**	Malta
CZE	Czech Republic	**NLD**	The Netherlands

Acronym	Detail		
DNK	Denmark	**POL**	Poland
EST	Estonia	**PRT**	Portugal
FIN	Finland	**ROM**	Romania
FRA	France	**SVK**	Slovakia
DEU	Germany	**SVN**	Slovenia
GRC	Greece	**ESP**	Spain
HUN	Hungary	**SWE**	Sweden
IRL	Ireland	**GBR**	United Kingdom

RESUMO

O objetivo desta tese é enfatizar a importância de uma abordagem multicamada numa análise de redes, através da aplicação das métricas mais adequadas a dois casos práticos: uma análise territorial tendo por base um conjunto de variáveis que caracterizam o sistema urbano português e um estudo dos mais importantes e influentes países da União Europeia.

Por um lado, apresentando as bases teóricas para uma análise de redes monocamadas e multicamadas, o investigador desenvolve uma sólida seleção de conceitos e medidas analíticas, tendo em conta a estrutura geral dos diferentes tipos de redes. Por outro lado, o enquadramento teórico é testado em duas redes multicamada reais, de modo a mostrar a validade e significância das medidas introduzidas, capazes de extrair informação importante e não-aleatória sobre tais fenómenos complexos.

O autor analisa os (atuais) 308 municípios portugueses em alguns dos aspetos económicos que caracterizam uma sociedade (trabalho e educação) e os atuais 28 países que compõem a União Europeia em importantes prioridades de políticas de integração europeia tais como o comércio internacional, o investimento direto estrangeiro e os fluxos migratórios e de remessas.

Para ambos os cenários nacional e europeu, depois de uma análise individual a cada variável, o método mais adequado para detetar comunidades é implementado numa perspetiva multicamada. Tais resultados são seguidamente comparados com os inicialmente obtidos. A aplicação de redes multivariadas permite inferir padrões espaciais mais consistentes quando comparados com a estrutura geográfica obtida estudando cada camada de rede separadamente. Assim, o problema de investigação adjacente a esta dissertação contribui com um modelo de análise estruturado para identificar medidas analíticas apropriadas e espelhar as estruturas gerais do sistema urbano português e da União Europeia.

PALAVRAS-CHAVE: Análise de Redes, Extração de Conhecimento de Redes Multicamada, Deteção de Comunidades, Redes Territoriais

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CHAPTER 1 – INTRODUCTION

For reader's convenience, the thesis presents an introductory and contextualization chapter. This section contains a brief description of the main purpose of this project: personal motivation, goals to be accomplished, thesis organization and transversal information about the methodology approach.

1.1. MOTIVATION

Everything seems to be connected to everything else. Recently, there has been a growing public fascination with the complex interaction of modern society, following the intense growth of the Internet in the ease with which global communication now takes place. This is a phenomenon that involves networks, and the aggregate behaviour of groups of people, companies, countries and others. Such phenomena are based on links, the way things are connected, and the ways in which decisions can have refined consequences for the outcomes of everything else.

"Network analysis is based on the intuitive notion that patterns are important features of the lives of the individuals who display them. Network analysts believe that how an individual lives depends in large part on how that individual is tied into the larger web of social connections. Many believe, moreover, that the success or failure of societies and organizations often depends on the patterning of their internal structure" (Freeman, n.d.[1]).

Network analysis crosses from graph theory mathematics, social network theory, sociology and, more recently, in the past 10/15 years from several developments in fields as diverse as computer science, physics, biology or economics. Considering its importance for describing and analysing complex and structural systems, it is a topic with increasing implications in the economy and society, and it might constitute an important measurement tool for social policies of welfare issues.

[1] Available at: http://www.insna.org/what_is_sna.html

1.1. RESEARCH PURPOSE

This dissertation aims at reviewing the most recent techniques encountered on the literature about mono and multi-layer network analysis, followed by the application of the most adequate method to extract useful knowledge (centrality measures, clusters, patterns, among others) in two structured and real datasets, by particularly focusing in the study of the Portugal municipalities movements and by analysing important measures which reflects Europe's strengthen and competitiveness. The idea is to represent multi-flows between municipalities/countries based upon a relational logic, more than the usual quantitative. The results obtained from a multilayer analysis perspective shall provide sufficient elements to clearly understand how municipalities/countries are related among them, their dependency and importance. Thus, this thesis scope contributes to expand the space for a multilayer network analysis when describing a global society.

Using the obtained results of the multilayer analysis as basis to constitute a target attribute, together with additional selected data, the researcher further proposes to implement predictive learning models to accurately process patterns. Such mining data job shall allow deriving relevant information of how a municipality/country shall look like to be considered centred in a network.

1.2. OBJECTIVES

The objectives of the research are to:

- Apply the most recent network techniques to both the Portuguese society and EU-28 countries scenarios, using relevant data and economic issues;
- Detect communities of communities to identify the most economically attractive cities from Portugal and the leading countries of European Union;
- Find (centrality) metrics for mining multivariate analysis of the obtained networks;
- Compare individual variables outcomes and perform multi-layered analysis through network measures;
- Understand the relationship between the Portuguese municipalities and the EU countries;

- Discuss the most noticeable outcomes;
- Monitor key performance indicators to assess the municipalities and EU countries outputs;
- Apply learning models by predicting the classification of the Portuguese municipalities / EU-28 countries.

1.3. STRUCTURE

The structure of the research and report is indicated as follows (see **FIGURE 1** below). In Section 1, the researcher introduces the intended purpose and objective by writing and developing the topic. They also describe which methodology is used to implement what it is initially proposed to be achieved.

Theoretical background in mono and multi-layered networks is later examined in the social network analysis literature review – chapter 2 –, which includes some historic landmarks of the field and it presents the main definitions and instruments devoted to graph theory and its continuous approaches. Section 3 consists of data and model implementation: it describes different data sources; it includes a brief description of the performed pre-processing work to each initial collected database; it analysis the variables of both case studies individually and then it implements the multi-layered networks analysis based on similarities between municipalities/countries (multiplex visualization). In section 4, the results are discussed, particularly the multi-layered network synthesis. Conclusions, research implications, limitations and future research recommendations are described as well.

The thesis is organized in a sequential form in terms of pagination, numeration of topics, tables and figures (by chapters). Bibliographic references are compiled at the end.

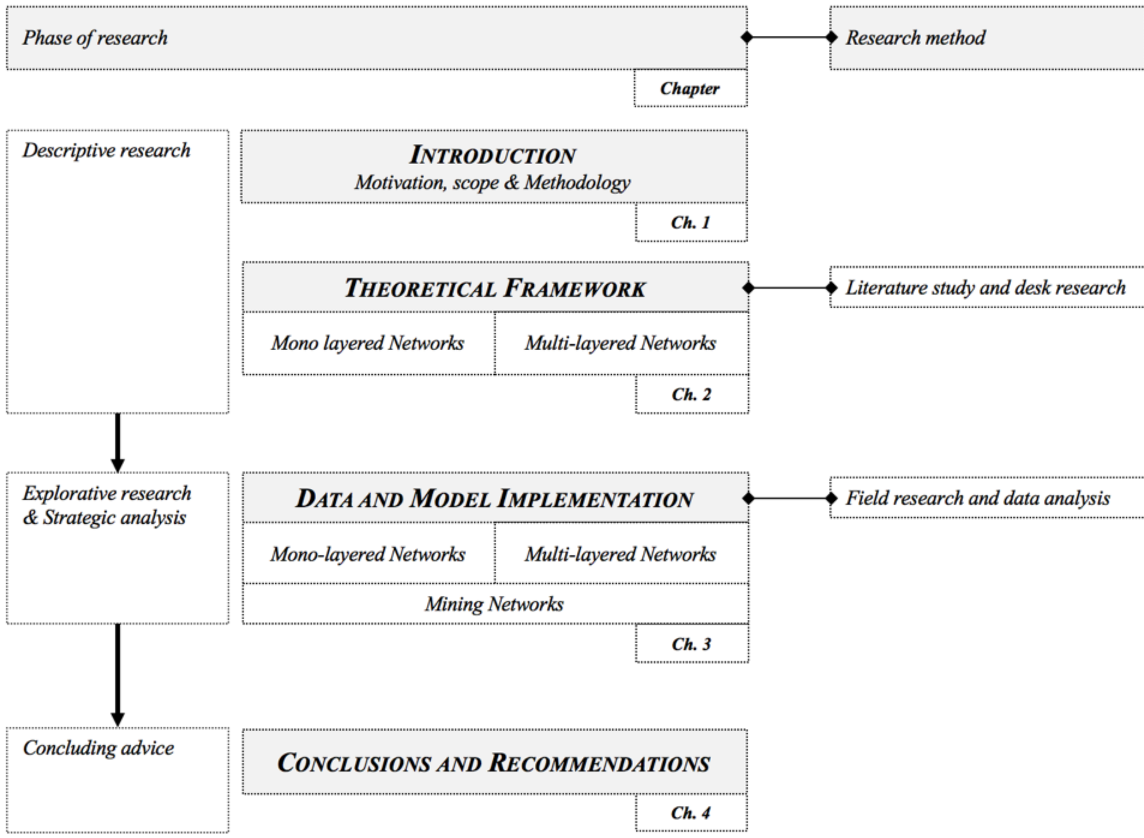


FIGURE 1 – Structure of the dissertation **SOURCE:** Author

1.4. METHODOLOGY APPROACH

The research method chosen to achieve the above-mentioned objectives of the dissertation is quantitative, based on data collected through information available in different sources: literature study – desk research – field research – data analysis (see **FIGURE 1** above). Two case studies aim to implement this methodology approach: on the one hand, the Portuguese municipalities are selected to provide a description of the Portuguese society; on the other hand, a second case study is conducted by considering the EU-28 countries, where different data contributes as detailed input to emphasise the economic landscape of the European Union. Two variables are considered for the Portuguese municipalities' case study: commuting interactions due to professional reasons and commuting interactions due to academic reasons. Four variables are selected for the analysis of EU countries: imports and exports interactions, investment relationships and remittance flows. For each topic, territorial networks and other network/statistical metrics are obtained using appropriate programs such as *R*[2], *Muxviz*[3], *ArcGIS* [4]and *Excel*. Several packages provided by each software are properly used. In particular, for *R*, igraph package (Version 1.0.1, June 26, 2015) is used to describe some routines of each individual network analysis as well as tnet (Version 3.0.14, November 18, 2015), providing the needed functions for the analysis of weighted networks. *Notepad++* was also a complementary used tool. The analysis includes centrality measures (degree, betweenness, closeness and eigenvector) as well as community detection, reciprocity and the respective territorial networks connecting municipalities/countries (using ArcGIS) are firstly computed in a monoplex perspective. Then, the *Muxviz* framework is used for the multilayer analysis and visualization of networks, allowing the creation of interactive visualization and exploration of multilayer networks. Final predictive analytical models are performed to municipalities / countries.

This approach seeks to discover the common relationships across the municipalities/countries. As such, this type of methodology seems to be the most adequate one to analyse the national and European geographical behaviours, and formulate a solution to the research problem of this dissertation.

[2] Available at cran.r-project.org
[3] Available at muxviz.net
[4] Available at arcgis.com/features/index.html

CHAPTER 2 – RELATED WORK | SOCIAL NETWORK ANALYSIS

Chapter 2 introduces the research background. Empirical concepts behind the graph theory and the study of network structure are outlined, including the historical background and the revision of a variety of useful quantities or measures that capture several features of the network topology/structure.

2.1. HISTORICAL BACKGROUND

The term social network analysis (SNA) was first used in the beginning of 20^{th} century by the sociologists Georg Simmel and Émile Durkheim (Freeman, 2004). They enhanced the importance of studying and understanding the social actors' interactions gathering patterns of the relationships between them.

Since then, several other social scientists started using the concept of "social networks" to denote complex sets of relationships between members of social systems at all scales, from interpersonal to international. Moreover, it emerged, not once, but several times in several different social science fields and in several places. In the 1930s, the psychiatrist *Jacob Moreno* and the sociologist *Helen Jennings* introduced basic analytical methods: firstly, among the inmates of a prison (Moreno, 1932, cf. Scott and Carrington, 2011) and later among the residents in a reform school for girls (Moreno, 1934, cf. Scott and Carrington, 2011). They named their approach *sociometry*. At first, sociometry, as a measure of the "socius" (the interpersonal connection between two people), generated a great deal of interest, particularly among American psychologists and sociologists. Nevertheless, by the 1940s, most of the American social scientists had returned to their traditional focus on the characteristics of individuals. Nevertheless, *W. Lloyd Warner* adopted a social networks approach (Freeman, 2004), when created the "bank wiring room" study, a social network component of the Western Electric research on industrial productivity (Roethlisberger and Dixon, 1939).

Another version of social network analysis emerged by the hand of a German psychologist, *Kurt Lewin*, in the University of Iowa in 1936 (Freeman, 2004). Mostly with graduate and post-docs students, a structural perspective was developed and conducted within social network research in the field of social psychology (e. g. Lewin and Lippit, 1938). One of Lewin's students (Alex Bavelas) led a famous study of the impact of group structure in productivity and morale. Despite all the completed research in the field until there, none approach was considered good enough to be accepted across all the social sciences in all countries; none provided a standard for structural research.

TABLE 1 lists some of the most well-known social network centres emerged during the period 1940-1969 all over the world.

TEAM LEADERS	FIELD	PLACE	COUNTRY
Charles P. Loomis Leo Katz	Rural Sociology	Michigan State	USA
Claude Lévi-Strauss André Weil	Linguistics	Sorbonne	France
Thorsten Hägerstrand	Geography	Lund	Sweden
Nicolas Rashevsky	Mathematical Biology	Chicago	USA
Paul Lazersfeld Robert Merton	Sociology	Columbia	USA
Everett Rogers	Communication	Iowa State	USA
Max Gluckman	Sociology	Manchester	Great Britain
Ithiel de Sola Pool Manfred Kochen	Political Science	MIT	USA
Linton C. Freeman Morris H. Sunshine	Community Power	Syracuse	USA
Claude Flament	Psychology	Sorbonne	France
Edward Laumann	Sociology	Michigan	USA
Peter Blau James A. Davis	Sociology	Chicago	USA
Robert Mokken	Sociology	Amsterdam	Netherlands

TABLE 1 – Centres of social network research from 1940 to 1969

***SOURCE*:** Freeman, 2008, cf. Scott, J., Carrington P. J., 2011, pp. 27

By the 1970s, with the work of *Harrison C. White*, the social network analysis came to be widely recognized among the social scientists as a field of research. One of the most important contributors by then came from one of the White' students, the American sociologist and later professor at the Stanford University, *Marks. Granovetter* (Freeman, 2008, cf. Scott, J., Carrington P. J., 2011). He is particularly known by his public paper "*The Strength of Weak Ties*", providing "a fragment of a theory", an "exploratory and programmatic" model, emphasizing "the personal experience of individuals is closely bound up with larger-scale aspects of social structure, well beyond the purview or control of particular individuals" (Granovetter, 1973). He also studied how people got jobs and discovered that they were more likely to get them through acquaintances than through friends. As such, he believes both strong and weak ties should be considered: weak ties as indispensable to individuals' opportunities to their integration into communities; strong ties as a part of the overall fragmentation. *Granovetter* pointed demography, coalition structure and mobility as important areas to be developed a micro-macro linkage with the help of network analysis.

In the late 1990s, however, there was a revolutionary change in the field. Even if the physicists were claiming their topics as research in physics, the topics typically studied in the social network analysis field were embraced, as well as the same structural perspective. At the same time, the physicists succeeded in getting biologists and computer scientists involved. Since then, the field has become a very active research area and the impact was then to produce a revolution in the social network research (Freeman, 2004).

FIGURE 2 shows the number of articles published on centrality by field of research over the second half of the twentieth century. It provides sufficient evidence to note that once the physicists and biologists started publishing in their area of expertise, they quickly overtook the social networks analysts.

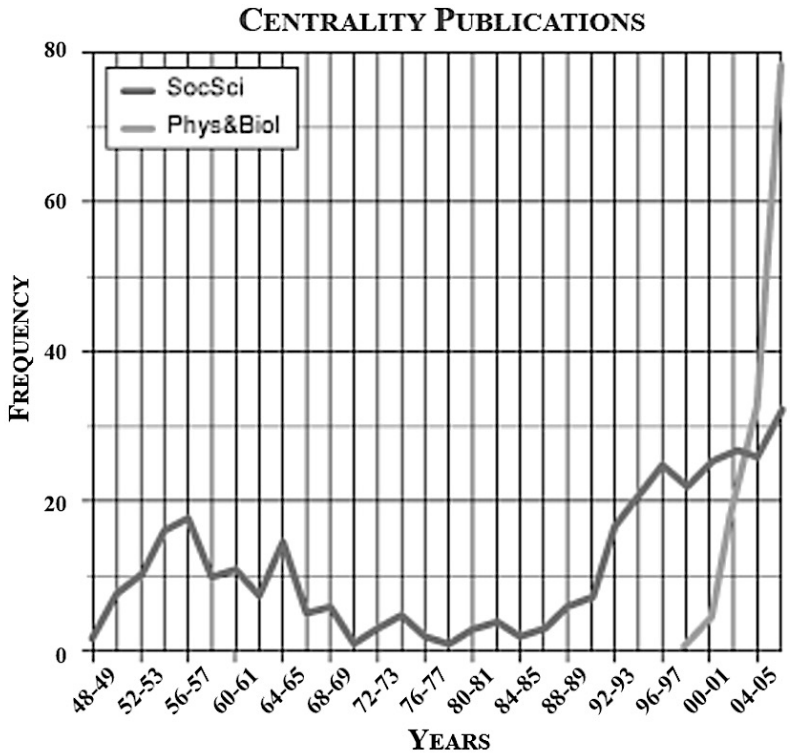


FIGURE 2 – Articles on centrality by date and field **SOURCE:** Freeman, 2008, cf. Scott, J., Carrington P. J., 2011, pp. 34, 35

Two important issues for the networks field, arising in the late 90ies, comprise the work done by Watts and Strogatz (1998) about cohesive groups – "small worlds" and the study of the distribution of degree centralities, proposed by Barabási and Albert (1999).

The goal of the work of *Duncan Watts* and *Stevan H. Strogatz* ("small worlds") was to find patterns of acquaintanceship linking pairs of persons. They concluded that a chain of acquaintanceships involving no more than seven intermediaries links any two people in the United States. The Watts and Strogatz model began with an attempt to capture clustering – the universal tendency of friends of friends to be friends. They represented links among individuals as a circular framework (see **FIGURE 3**), where each node is an individual and each edge is a social link connecting two individuals. The two circular frameworks shown in **FIGURE 3** are a good illustration of clustering: neighbours of neighbours are, for the most part, neighbours.

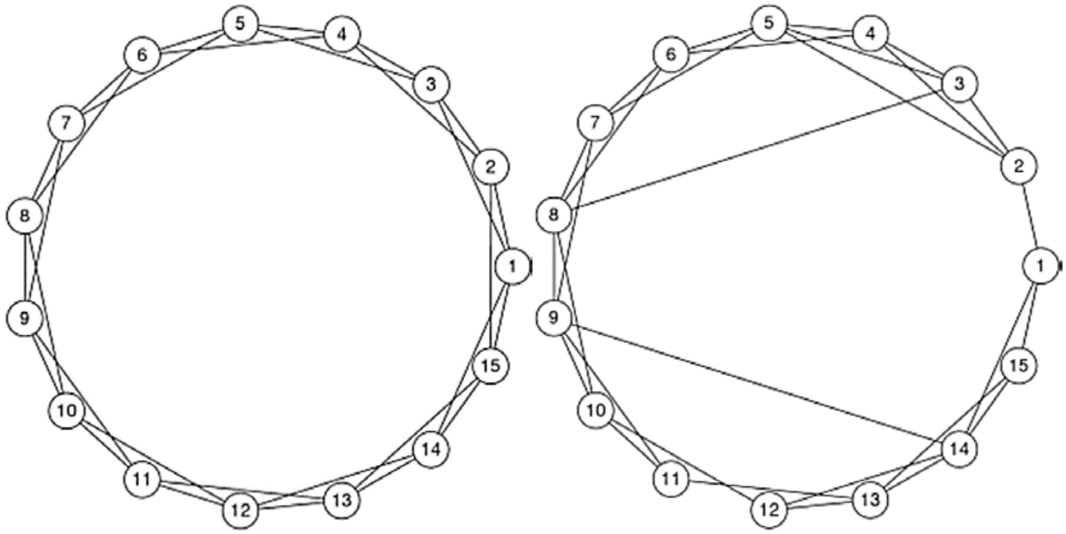


FIGURE 3 – High clustering frameworks with long and shorter path lengths, respectively

SOURCE: Freeman, 2008, cf. Scott, J., Carrington P. J., 2011, pp. 28, 29, 30

The average length of the path linking any two individuals is relatively larger in the first circular structure of **FIGURE 3**, when compared to the second lattice – that is the **small world effect**, where no individual is very far from any other individual. Watts and Strogatz could prove it simply by removing just a few of the links between close neighbours and replacing links to randomly selecting others. Thus, for the most part, friends of friends are still friends and the total world has become considerably smaller.

The article by Barabási and Albert (1999) also took up a standard network analytic topic, the degree distribution. Previous research in sociometry (Moreno and Jennings, 1938) reported the empirical result that the observed distribution of being chosen when people were asked whom they would choose, say, to invite to a party or to work with on a project, was considerable skewed. Barabási and Albert (1999) studied the distribution of connections in networks that grew as a consequence of adding new nodes, for instance in the links between sites in the World Wide Web, screen actors who worked together on films and links between generators, transformers and substations in the USA electrical power grid. Although Barabási and Albert were apparently unaware of the earlier findings of Moreno and Jennings (1938), they discovered that the connections in the examined networks they were not random (Freeman, 2011): a few nodes displayed too many connections and many nodes displayed too few.

2.2. MONO-LAYERED NETWORKS

a. GENERAL FORM & BASIC DEFINITIONS

Globally, a (mono-layered or monoplex) network is usually perceived as a collection of objects together with a set of ties that connect pairs of objects and represent the relationships between them. Examples of such relations include who is a friend with whom, or who is the supervisor of whom (Borgatti, 2008). Moreover, the terms "mono-layered" or "monoplex" indicate that nodes and edges are organized in one only single layer.

One can formally define a graph as $\boldsymbol{G} = (\mathcal{N}, \mathcal{E})$, consisting of the set $\mathcal{N}$ of nodes and the set $\mathcal{E}$ of edges, which are ordered pairs of elements of $\mathcal{N}$.

FIGURE 4 represents the structure of the Internet, where the nodes are the group of computers with similar IP addresses and the links are the routes taken by IP packets (usually optical fibre).

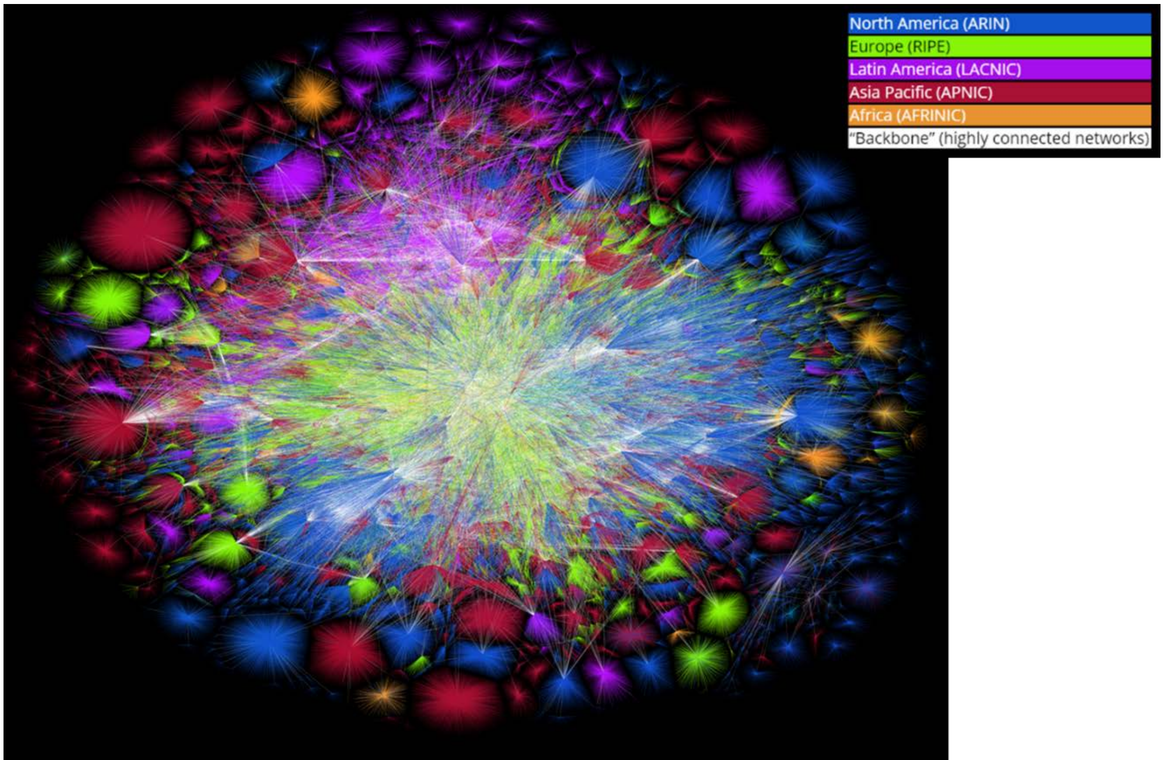


FIGURE 4 – The structure of the Internet (on July 11th, 2015) **SOURCE:** opte.org

Objects are typically called *nodes*, *vertices* or *actors* and the relationship between the objects are called *links* or *edges*. The information associated with the objects and relationships are called *attributes*, *features*, *dimensions* or *properties*. If there is an edge between a pair of nodes, then this edge is incident to each of the two nodes and these nodes are adjacent to each other. Two edges that are incident to the same node are said to be “incident” to each other (Bollobás, 1998).

Networks may also have multiedges (repeated edges between the same pair of vertices), self-edges (edges connecting a vertex to itself), hyperedges (edges that connect more than two vertices together) and many other features (Newman, 2008).

In the scientific literature, the terms network and graph are used interchangeably.

	ENGINEERING AND COMPUTER SCIENCE	MATHEMATICS AND GRAPH THEORY	PHYSICS	SOCIAL SCIENCES
“Point”	Node	Vertex	Site	Actor
“Line”	Link	Edge	Bond	Tie
“Network”	Network	Graph	Network	Network

TABLE 2 – Different expressions for each field of nodes and links **SOURCE:** Gastner, M. T. (2011), pp 1

A network can either be:

- simple or multigraph: in a simple network, there cannot be multiple links (multiedge) between two nodes and no node can be connected to itself (i.e. no self-loops); if, on the other hand, multiple links are allowed, then the network is called a multigraph.
- directed or undirected: when the order of links does not matter, then a network is called undirected; if links only go in one direction, a network is entitled directed or digraph.

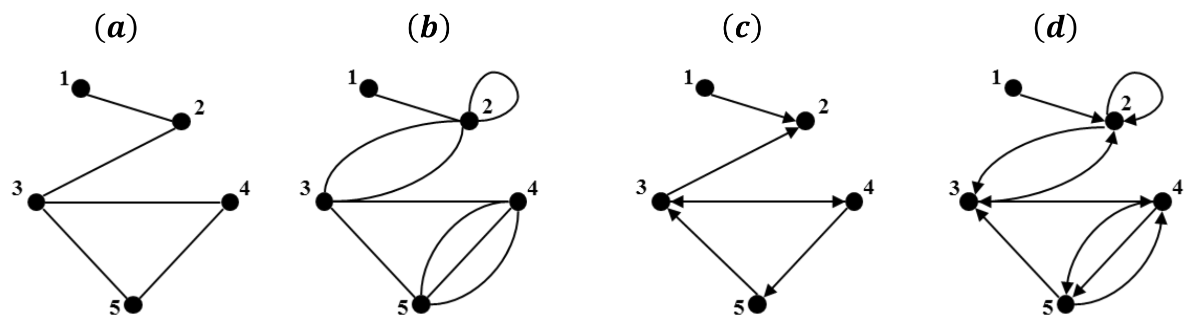


FIGURE 5 – Examples of different types of networks **SOURCE:** Author

FIGURE 5 shows examples of each possible type of network:

- $(\boldsymbol{a})$ is a simple undirected graph. It can be represented by specifying the number of nodes n and its edge list as follows: $n = 5$ and the links are $(1,2),(2,3),(3,4),(3,5)$ and $(4,5)$
- $(\boldsymbol{b})$ represents an undirected multigraph;
- $(\boldsymbol{c})$ corresponds to a simple directed network;
- $(\boldsymbol{d})$ is a directed multigraph.

A network is considered homogeneous if all the nodes and links are of the same type, such as a friend or a web page network. On the other hand, a network is heterogeneous if the nodes and links are of different types, such as publication networks (linking together authors, conferences, papers, and contents), and health-care networks (linking together doctors, nurses, patients, diseases, and treatments).

Another way to represent a network is by drawing an adjacency matrix. The adjacency matrix A of a simple network is the matrix with elements $\boldsymbol{A_{ij}}$ such that:

$$\boldsymbol{A_{ij}} = \begin{cases} 1\ if\ there\ is\ a\ link\ from\ j\ to\ i, \\ 0\ otherwise. \end{cases}$$

Given the same examples of **FIGURE 5**, the respective adjacency matrixes are as follows:

$$\underset{(\boldsymbol{a})}{\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}} \quad \underset{(\boldsymbol{b})}{\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}} \quad \underset{(\boldsymbol{c})}{\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}} \quad \underset{(\boldsymbol{d})}{\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}}$$

It can be noticed that the elements $\boldsymbol{A_{ii}}$ of the diagonal lines of the simple networks ($(\boldsymbol{a})$ and $(\boldsymbol{c})$) are null (no self-loops). Also, undirected networks ($(\boldsymbol{a})$ and $(\boldsymbol{b})$) are both symmetric once if there is a link between i and j, then there is also a link between j and i). Directed networks ($(\boldsymbol{c})$ and $(\boldsymbol{d})$) are asymmetric.

b. Bipartite Networks

A bipartite graph (or bigraph) is a network whose nodes can be divided into two disjoint sets U and V, such that each link connects a U-node to a V-node (Barabási, 2016). Nodes in the U-set connect directly only to nodes in the V-set. Hence there are no direct U-U or V-V links.

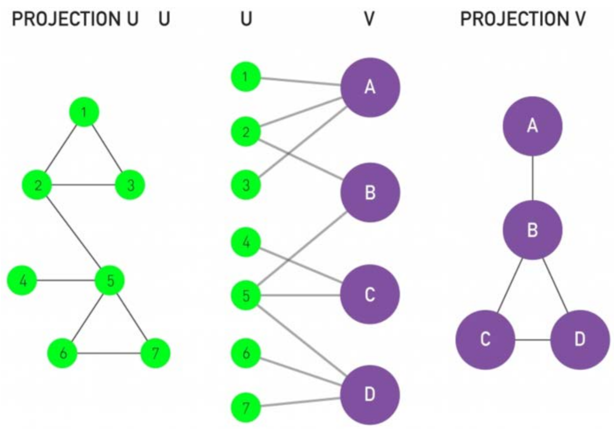


Figure 6 – Representation and projections of bipartite networks **Source:** Barabási, 2016

The previous figure shows the two projections that can be generated from any bipartite network. Projection U is obtained by connecting two U-nodes by a link if they are linked to the same V-node in the bipartite representation. Projection V is obtained by connecting two V-nodes to each other if they connect to the same U-node in the bipartite network.

A well-known example of a bipartite networks is the Hollywood actor network, in which one set of nodes corresponds to movies (U), and the other to actors (V). A movie is connected to an actor if the actor plays in that movie.

c. Weighted Networks

So far, the discussion was mostly focused on unweighted networks, i.e. networks that have a binary nature, where the edges between nodes are just either present or absent (coded respectively as 0 or 1). Nevertheless, along with a complex topological structure, many real networks display a large heterogeneity in the capacity and the intensity of the connections. Going beyond purely topological models, the richness and complexity of real systems like the unequal traffic on the Internet or the uneven fluxes of passengers in an airline networks can be better explained in terms of weighted networks, i.e. networks in which links have some form of weight enclosed to them, each one of them carries a numerical value measuring the strength of the connection (Boccaletti et al., 2006). In a social network, the weight of a tie is generally a function of duration, emotional intensity, intimacy, and exchange of services (Granovetter, 1973).

d. WALKS, CYCLES & PATHS

Walks and paths – and their lengths – are important concepts in both graph theory and network science. A **walk** is simply a sequence of nodes $v_1 \rightarrow v_2 \rightarrow \cdots \rightarrow v_k$ in which each consecutive pair of nodes in the sequence is connected by an edge in the network.

A **cycle** represents informally a "ring" structure. It is a walk with at least three edges that begins and ends at the same node (i.e. $v_1 = v_k$). In concrete, cycles in communication and transportation networks often present the alternate routings that go the "other way" around the cycle. In the social network of friendships, cycles can be noticed in a daily basis. If, for example, one wife's cousin's close friend is in fact someone who works with his brother, this is a cycle – consisting of that person, his wife, her cousin, his friend, his co-worker (i.e. his brother), and finally back to that person.

A **path** in a graph is a walk that does not contain any cycles.

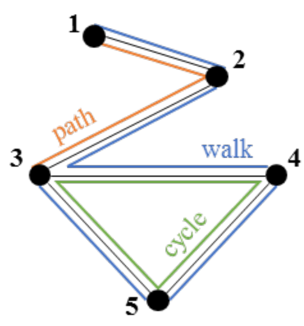


FIGURE 7 – Definitions of walk, cycle and path in a network **SOURCE:** Author

Observing the example given in **FIGURE 7**, the sequence of nodes *1 2 3 4 5 3* represents a walk; sequence of nodes *3 4 5 3* forms a cycle; the sequence of nodes *3 2 1* forms a path.

When edges are weighted, the corresponding length of a walk (cycle, path) is measured as the sum of the values of the weights along the edges traversed in the walk (cycle, path).

e. GIANT COMPONENTS & CONNECTIVITY

A graph is connected if for every pair of nodes, there is a path between them.

FIGURE 7 is connected; in general, for most communication and transportation networks, it is expectable them to have connectivity since their goal is to move traffic from one node to another. Nevertheless, there is a priori no reason to expect graphs to be necessarily connected: case of social networks, where it may not normally be possible to construct a path from one person to the other.

Moreover, if a graph is not connected, then it breaks apart into a set of connected components so that each subgroup of nodes is connected when considered as a graph in isolation.

It is usual to say that a connected component of a graph is a subset of the nodes such that:

- **(i)** every node in the subset has a path to every other (cycle); and
- **(ii)** The subset is not part of some larger set with the property that every node can reach every other.

Both (i) and (ii) are necessary to formalize the intuitive definition: (i) says that the component is indeed internally connected, and (ii) says that it really is a free-standing "piece" of the graph, not a connected part of a larger piece.

Giant components are a useful qualitative way of thinking about the connected components of typical large networks (Kivelä et al., 2014). Considering the hypothetical social network of the entire world, it might include:

- **(i)** Some independent components: for instance, a single person with no living friends would constitute a one-node component in the global friendship network. Or the canonical "remote tropical island," consisting of people who have had no contact with the outside world, would also be a small component in the network;

(ii) Some giant component: imagining a person who have friends from other countries, that person would have the same component as all these friends. Considering also the parents of these friends, their friends and descendants, then all of these people would be in the same component as well.

So even though the global friendship network may not be connected, the giant component of the social network of the entire world seems to be very large indeed – it reaches into most parts of the world, includes people from many different backgrounds, therefore it seems likely to contain a significant fraction of the world's population.

This is particularly true when one looks across a range of network datasets – large, complex networks[5] often have what is called a giant component. Moreover, a giant component is generally characterized by its uniqueness: when a network contains a giant component, normally it contains only one.

f. Centrality Measures

Analysing a graph in terms of its components, its densely-connected regions and the boundaries between them, is a powerful way of describing the network structure. However, within a given component, there might even be richer internal structure that is important to the network's interpretation. One way to formalize the role of the prominent central node is to observe if the largest connected component would break apart into distinct components if that node would be removed.

Over the years, centrality has become one of the most important and widely used conceptual tools for analysing networks (Borgatti and Everett, 2003) and, in particular, to identify the most important nodes within a graph. Network researchers have introduced a large number of centrality indices, defined on the vertices of the graph according to one criterion or another (Newman, 2005 in Wasserman and Faust, 1994; Scott, 2000). Four different measures of centrality are presented as follows:

[5] A complex network contains non-trivial topological features; it contains many different subgraphs (Kim, J., Wilhelm, T., 2008)

- The **degree** informs how many neighbours a node have. In other words, it gives the basic connectedness of a node;
- The main concept behind the **betweenness** is to assess how important a node is as an intermediary;
- The **closeness** measures the ease with which a node is close to another, or how easily it can access other nodes;
- The **eigenvector** centrality identifies the influence and prestige of a node, by capturing the value of its neighbours.

f.1. DEGREE (CONNECTEDNESS)

Degree is the simplest of the node centrality measures by only using the local structure around nodes. In a simple and undirected network, the degree of a node corresponds to the number of other nodes to which it is directly connected by edges (i.e., the number of its immediate neighbours). The degree centrality can be computed from the adjacency matrix, i.e. $\boldsymbol{k_i} = \sum_{j=1}^{n} A_{ij}$ (see section 2.2.a). In a directed network, it is usual to distinguish between in-degree $\left(k_i^{in}\right)$ of a node – number of ingoing links – and out-degree (k_i^{out}) – number of outgoing links. From the adjacency matrix, $\boldsymbol{k_i^{in}} = \sum_{j=1}^{n} A_{ij}$ and $\boldsymbol{k_i^{out}} = \sum_{i=1}^{n} A_{ij}$. In **FIGURE 5**, the node 3 from network $(\boldsymbol{a})$ has degree 3 and the same node 3 of network $(\boldsymbol{c})$ has in-degree 2 and out-degree 1.

f.1.1. DEGREE IN WEIGHTED NETWORKS

In a weighted graph, the natural generalization of degree $\boldsymbol{k_i}$ of a node i has been extended to *node strength* (or node weight, or node weighted connectivity) $\boldsymbol{s_i}$, defined as the sum of weights (Barrat et al., 2004; Newman, 2004): $\boldsymbol{s_i} = \sum_{j \in \mathrm{N}} w_{ij}$. Although it is the preferred measure for analysing weighted networks (Barrat et al., 2004; Opsahl et al., 2008), by considering only the total level of involvement in the network, node strength fails to consider the number of ties, original measure formalized by Freeman (1978). In an attempt to combine both degree and strength, Opsahl et al. (2010) used a tuning parameter to set the relative importance of the number of ties compared to tie weights. There are two benchmark values for the tuning parameter (0 and 1).

- If the parameter is set to the benchmark value of 0, the outcomes of the measures are solely based on the number of ties. This application corresponds to the measure found and suggested by Freeman (1978);
- Conversely, if the value of the parameter is 1, the measure is based on tie weights only – proposed generalization of Barrat et al. (2004).

f.1.2. DEGREE DISTRIBUTIONS

In an undirected network, the degree distribution is the sequence $p_1, p_2, \ldots, p_k$, where p_k is the fraction of nodes in the network with degree k. The distributions are often "heavy-tailed": there are some nodes ("hubs") with very high degree. Taking into account **FIGURE 5** $(\boldsymbol{a})$, the following degree distributions can be denoted: $p_1 = \frac{1}{5}, p_2 = \frac{3}{5}, p_3 = \frac{1}{5}$.

Similarly, it can be defined in-degree distribution and out-degree distribution in a directed network. Considering **FIGURE 5** $(\boldsymbol{c})$, $p_0^{in} = \frac{1}{5}, p_1^{in} = \frac{2}{5}, p_2^{in} = \frac{2}{5}$; $p_0^{out} = \frac{1}{5}, p_1^{out} = \frac{2}{5}, p_2^{out} = \frac{2}{5}$.

f.2. BETWEENNESS (INTERMEDIARY ROLE)

Another important concept of centrality measures is the class of betweenness, which mostly determines the extent to which a vertex lies on paths between other vertices. The idea was originally accredited by Freeman (1977). Betweenness centrality of a node reflects the amount of control that this node exerts over the interactions of other nodes in the network (Newman, 2010). It is an approximate guide to the influence that the vertices have over the network between others. Vertices with high betweenness centrality may have considerable influence within a network. This measure also favours nodes that join communities (dense subnetworks), rather than nodes that lie inside a community. Considering the set of all geodesic paths[6] in a network, the betweenness centrality can be built by, mathematically, letting n_{st}^{i} be a binary function as described below:

$$n_{st}^{i} = \begin{cases} 1, if\ i\ lies\ on\ the\ geodesic\ path\ from\ s\ to\ t \\ 0\ otherwise \end{cases}$$

Then, the absolute betweenness centrality x_i is given by $\boldsymbol{x_i} = \sum_{st} n_{st}^{i}$. This definition counts separately the geodesic paths in either direction between each vertex pair – for undirected networks each path is indeed counted twice. For that reason, some approaches of the betweenness centrality compensate for this by dividing x_i by 2. Other variant excludes paths from each vertex to itself, so that $x_i = \sum_{s \neq t} n_{st}^{i}$ (this definition simply decreases the betweenness by 1). Many authors agree that the initial equation has the advantage that it can be applied unmodified to directed networks; moreover, in practice it makes no difference whether dividing centrality by 2, since the focus is the relative magnitudes of the centralities and not their absolute values. Formally, redefining n_{st}^{i} as the number of geodesic paths from s to t that pass through i and given g_{st} as the total number of geodesic paths from s to t, betweenness centrality of vertex i (both for undirected and directed networks) can be expressed as follows:

$$\boldsymbol{x_i} = \sum_{st} \frac{n_{st}^{i}}{g_{st}}$$

Betweenness centrality differs from the other centrality measures in being not principally a measure of how well-connected a vertex is. Instead, it measures how much a vertex falls "between" others (Newman, 2010). Indeed, a vertex can have low degree, be connected to others that have low degree, and still have high betweenness. This normally happens for a vertex that lies on a bridge joining two groups of other vertices.

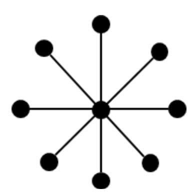


FIGURE 8 – Example of a *star graph* **SOURCE:** Author

[6] Geodesic path characterizes the shortest path through a network between two vertices.

Betweenness centrality values are typically distributed over a wide range. The maximum possible value for the betweenness of a vertex occurs for the central vertex in a *star graph* (network composed of a vertex attached to $n-1$ others by single edges): the central vertex lies on the shortest path between every other pair of vertices. In this situation, the central vertex lies on all n^2 shortest paths between vertex pairs except for the $n-1$ paths from the peripheral vertices to themselves. As such, the betweenness centrality of the central vertex is $n^2 - n + 1$.

On the other end of the scale, the smallest possible value of betweenness in a network with a single component is $2n-1$, since at a minimum each vertex lies on every path that starts or ends with itself. This situation occurs, for instance, when a network has a "leaf" attached to it, a vertex connected to the rest of the network by just a single edge. The ratio between the largest and the smallest possible betweenness values is then given by: $\frac{n^2-n+1}{2n-1} \approx \frac{n}{2}$

So far, the considered betweenness values for the computations are merely raw path counts; in some cases, it is convenient to normalize betweenness. One natural way is to normalize the path count by dividing by the total number of (ordered) vertex pairs, which is n^2, so that betweenness becomes the fraction (rather than the number) of paths that run through a given vertex:

$$\boldsymbol{x_i} = \frac{1}{n^2}\sum_{st}\frac{n_{st}^i}{g_{st}}$$

With this definition, the values of the betweenness lie strictly between zero and one.

Betweenness measure has still other several broad general frameworks, based on different models of diffusion, transmission, or flow along network edges.

f.2.1. Random-Walk Betweenness

In this betweenness variant, proposed by Newman (2005), the traffic between vertices s and t is thought of as performing a random walk that starts at vertex s and continues until it reaches vertex t. The betweenness is defined according to $\boldsymbol{x_i} = \sum_{st} n_{st}^i$ but n_{st}^i is now defined as the number of times that the random walk from s to t passes through i on its journey, averaged over many repetitions of the walk. In this case $n_{st}^i \neq n_{ts}^i$ in general, even on an undirected network.

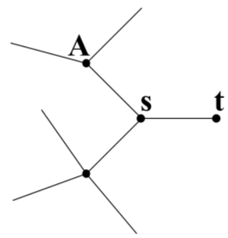


FIGURE 9 – Portion of a given network **SOURCE:** Author

A random walk from s to t may pass through vertex A before returning to s and stepping thenceforth to t; but a walk from t to s will never pass through A because its first step away from t will always take it to s and then the walk will finish. Since every possible path from s to t occurs in a random walk with some probability, the random-walk betweenness includes contributions from all paths.

Random walk betweenness is an appropriate betweenness measure for traffic that crosses a network with no idea of where it is going – it simply walks around at random until it reaches its destination (Newman, 2005).

f.2.2. BETWEENNESS IN WEIGHTED NETWORKS

Brandes (2001) proposed a new algorithm allowing betweenness scores to be calculated for nodes on weighted networks. Brandes based his generalisation in Freeman's research (1978); however, he focused solely on tie weights, instead of the original feature considering the number of ties.

A second approach proposed by Opsahl et al. (2010) stablishes a bridge between Freeman and Brandes generalization's, by incorporating both the number of ties and the tie weights. Opsahl et al. introduced a tuning parameter (alpha) to set the relative importance of the number of ties compared to tie weights. There are two benchmark values for the tuning parameter:

- If alpha is set to 0, the outcomes of the measures are uniquely based on the number of ties, and are equal to the one found when applying Freeman's measure (1978);
- On the other hand, if alpha = 1, the measure is based on tie weights only, and are identical to the proposed generalisation of Brandes (2001).

If the parameter is set between the benchmark values (0 and 1), the existing measures are combined (Brandes, 2001; Freeman, 1978).

f.3. CLOSENESS (ACCESSIBILITY)

An entirely different measure of centrality is provided by the closeness centrality. It is defined as the inverse of the sum of distances to all other nodes from a focal node (Freeman, 1978) and it is based on the concept of mean geodesic path[7]. Considering d_{ij} as the length of a geodesic path from i to j (number of edges along the path), then the mean geodesic distance from i to j, averaged over all vertices j in the network, is given by $l_i = \frac{1}{n}\sum_i d_{ij}$. This measure takes low values for vertices that are separated from others by only a short geodesic distance on average. Some authors exclude vertex's influence on itself, so that $l_i = \frac{1}{n-1}\sum_{j(\neq i)} d_{ij}$, which might be a reasonable strategy, since the distance d_{ij} from i to itself is zero by definition, so this term in fact is not relevant to the sum. Nevertheless, the first equation tends to give slightly more precise analytic results (Newman, 2011).

The mean geodesic path l_i is *lower* for vertices that are more central in the sense of having a shorter network distance on average to other vertices. Therefore, it cannot be considered a centrality measure in the same sense as others. To overcome this shortcoming, researchers commonly calculate the inverse of l_i, allowing then to obtain the **closeness centrality** $\boldsymbol{C_i} = \frac{1}{l_i} = \frac{n}{\sum_i d_{ij}}$.

Closeness centrality measures the mean distance from a node to other nodes. One of the limitations of using closeness is the difficulty to distinguish between central and less central vertices: the values tend to be cramped together with the differences between adjacent values showing up only when examining the trailing digits. This means that even small fluctuations in the structure of the network can change the order of the values substantially (Newman, 2011). Another barrier is the lack of applicability to networks with disconnected components: two nodes

[7] Geodesic path characterizes the shortest path through a network between two vertices.

that belong to different components do not have a finite distance between them. Thus, closeness is generally restricted to nodes within the largest component of a network.

f.3.1. CLOSENESS IN WEIGHTED NETWORKS

Closeness has been generalised to weighted networks by Newman (2001), using Dijkstra's (1959) algorithm and Freeman's (1978) approach.

Mainly, to create a closeness measure, Newman (2001) transformed the positive weights into costs by inverting them (dividing 1 by the weight). Then, Newman (2001) applied Dijkstra's algorithm and found the least-costly paths among all nodes. The total cost of the paths from a node to all others was a measure of distance: a high distance was transformed into a low closeness, and a low distance was transformed into a high closeness.

Similarly to Barrat et al.'s (2004) generalisation of degree, Newman's (2001) generalised algorithm solely focuses on the sum of tie weights, and fails to consider the number of ties on paths. Opsahl et al. (2010) generalisation of shortest paths can be applied to determining the length of them.

f.4. EIGENVECTOR (VALUE OF NEIGHBOURHOOD)

The eigenvector centrality is another measure of a node' structural influence in a network, proportional to the structural importance of its connected neighbourhood. The eigenvector centrality thesis reads: *A node is important if it is linked to by other important nodes*.

For a given graph G with V number of vertices, let $A = (a_{v,t})$ be the adjacency matrix, where:

$$a_{v,t} = \begin{cases} 1, if \text{ vertex } v\ is\ linked\ to\ vertex\ t \\ 0\ otherwise \end{cases}.$$

The relative centrality score of vertex v can be defined as:

$x_v = \frac{1}{\lambda}\sum_{t\in M(v)} x_t$, where $M(v)$ is a set of the neighbours of v and λ is a constant. The neighbourhood summation may be extended to the summation over all vertices present in a network, as only connected vertices will contribute to the eigenvector centrality. Hence, $x_v = \frac{1}{\lambda}\sum_{t\in G} a_{v,t}x_t$.

By adopting the vector notation $x = (x_v)_{v\in V} \in \mathbb{R}^{|V|}$, the **eigenvector centrality** definition can be expressed in terms of the full adjacency matrix A: $\lambda x = Ax$. Given this expression, it is possible to understand that the eigenvector centrality of a node v is recursively proportional to the importance of its neighbours $N(v)$. In fact, the eigenvector centralities consider both direct and indirect connections between agents. Several computational methods can calculate eigenvectors, and thus eigenvector centralities. One of the most common methods is a recursively implemented algorithm called the power iteration.

A variant of eigenvector centrality is employed by the well-known Web search engine Google to rank Web pages (the score of a web page is proportional to the sum of the scores of the pages linked to it).

g. RECIPROCITY

The study of link reciprocity, or the tendency of vertex pairs to form mutual connections, has received an increasing attention in recent years (Squartini, T. et al, 2013). Among other things, reciprocity has been shown to be crucial to classify directed networks, understand the effects of the network structure on dynamical processes and explain patterns of growth in out-of-equilibrium networks (Garlaschelli, D. and Loffredo, M. 2008). “Reciprocity tells you how likely it is that a vertex that you point to also points back to you. In general, it’s found that you are much more likely to link to me if I link to you than if I don’t.” (Newman, 2011).

The measure of reciprocity defines the proportion of mutual connections. It is most commonly defined as the probability that the opposite counterpart of a directed edge is also included in the graph. In other words, it corresponds to the probability of mutual connection between a vertex pair, if it is known that there is a (possibly non-mutual) connection between them.

h. Community Detection

Due to the high complexity of a given graph, distribution of edges in a network might be not only globally, but also locally inhomogeneous, with high concentrations of edges within special groups of vertices, and low concentrations between these groups. Many real networks consist of modules which are densely connected themselves but sparsely connected to other modules. This feature is called community structure. Communities, also called clusters or modules, are groups of vertices which probably share common properties and/or play similar roles within the graph.

The aim of community detection in graphs is to identify the modules and, possibly, their hierarchical organization, by using the information encoded in the graph topology. Identifying modules and their boundaries allows the classification of vertices, according to their structural position in the modules. Therefore, vertices with a central position in their clusters, i.e. sharing a large number of edges with the other group partners, may have an important function of control and stability within the group; vertices lying at the boundaries between modules play an important role of mediation and lead the relationships and exchanges between different communities (Fortunato, 2010 cf Csermely, 2008).

Finding communities within an arbitrary network can be a computationally difficult task. However, several methods for community finding have been developed and employed with varying levels of success. Such algorithms mainly include:

- Minimum-cut method
- Hierarchical clustering
- Girvan–Newman algorithm
- Modularity maximization
- Clique-based methods

h.1. Walktrap

More than traditional partitioning methods, the general idea behind this dynamic algorithm is based in the intuition that random walks on a graph tend to get “trapped” into densely connected parts corresponding to communities (Pons, 2005). Walks are more likely to stay within the same community because there are only a few edges that lead outside a given community. Walktrap runs short random walks of a given number of steps (parameterized) and uses the results of these random walks to merge iteratively the vertices into communities.

2.3. MULTI-LAYERED NETWORKS

The social scientist John Barnes once described graph theory as a "terminological jungle, in which any newcomer may plant a tree" (Easley and Kleinberg, 2010). More recently, it has suddenly become very fashionable to study networks with multiple layers (or multiple types of edges), multiplex networks, interdependent networks, networks of networks and many others.

To represent networks at multiple levels or with multiple types of edges (or with other similar features), structures shall consider layers in addition to nodes and edges.

a. GENERAL FORM & BASIC DEFINITIONS

In the most general multilayer network framework, each node can belong to any subset of the layers; it is possible to consider edges that encompass pairwise connections between all possible combinations of nodes and layers. As such, any node $\boldsymbol{u}$ from layer $\boldsymbol{\alpha}$ can be connected to any node $\boldsymbol{v}$ in any layer $\boldsymbol{\beta}$. Moreover, it should also be considered "multidimensional" layer structures so it will be possible to include every type of multilayer network construction that have been encountered in the literature so far. For example, one "dimension" of a layer might be the type of an edge and another aspect might be the time at which an edge is presented. The above use of the word "dimension" had been standardly used in mathematics and physics to denote a layer in a multilayer network: in the social-networks literature, for instance, one might discuss different "dimensions" of interactions between people (friendship, family, work colleagues, etc.). "Dimension" or "elementary layer" is frequently used to denote an element, aspect or feature. The term "layer" refers to a combination of elementary layers from all aspects. The interaction type and a time stamp would be both examples of an elementary layer. **FIGURE 10** is an example of a multilayer network with two elementary layer structures ($\boldsymbol{L_1} = \{A;\ B;\ C\}$ and $\boldsymbol{L_2} = \{X;\ Y\}$) and six single layers or simply monoplex networks $\big((A;\ X), (A;\ Y), (B;\ X), (B;\ Y), (C;\ X), (C;\ Y)\big)$.

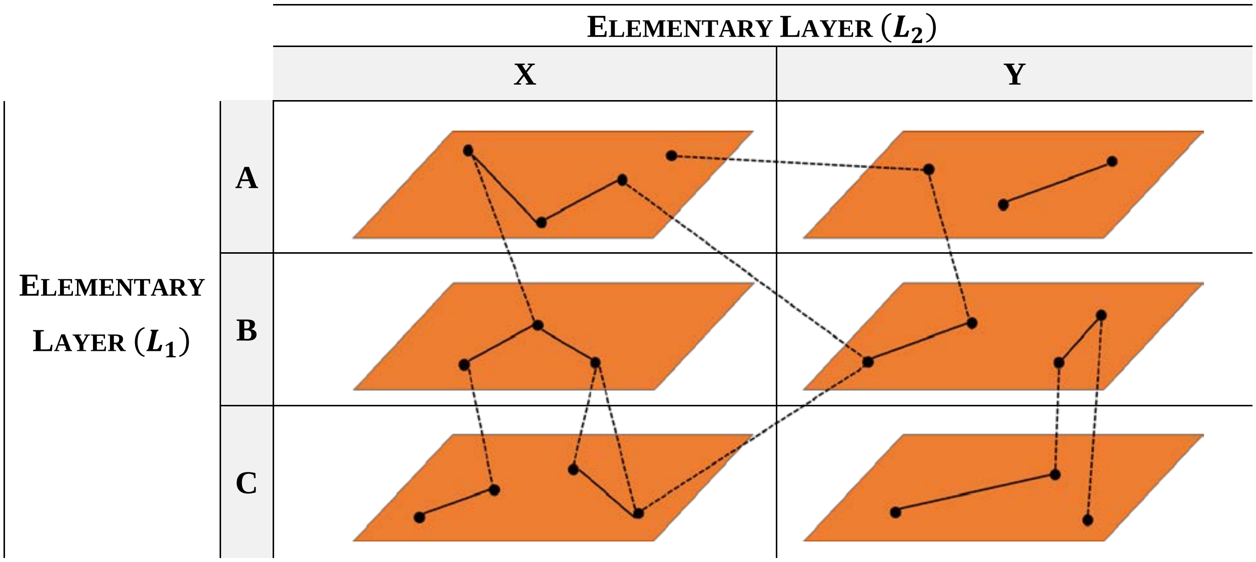


FIGURE 10 – Example of a multilayer network **SOURCE:** Author

Visualize **FIGURE 10** as a hospital hierarchical network can constitute a practical example of multilayer networks. In this sense, let us assume $\boldsymbol{L_1} = \{A = Doctors;\ B = Nurses;\ C = Patients\}$ and $\boldsymbol{L}_2 = \{X = Cardiology;\ Y = Neurology\})$. For instance, a given doctor from the cardiology unit can have several nurses at his/her disposal from both cardiology and neurology divisions. Similarly, each nurse can be responsible for taking care of several patients from different divisions as well. Doctors, nurses and patients can be assigned to both Cardiology/Neurology units (in this case the same node can be presented horizontally in each layer), and it could also be possible that, for a given moment in time, a doctor could become nurse and/or patient (in this case the same node can be depicted in each vertical layer).

In a multilayer network, as in a monoplex network, the term adjacency is used to describe a direct connection via an edge between a pair of node-layers and the term incidence describes the connection between a node-layer and an edge. Two edges that are incident to the same node-layer are also "incident" to each other. All the possible types of edges can occur between any pair of node-layers – including when a node is adjacent to a copy of itself in some other layer.

In terms of a graph representation, a multilayer network can be considered as a set of edges and nodes that are labelled in a certain way – nodes can be connected to each other in a pairwise manner both within the layers and across the layers. It is both typical and convenient to use different semantics for edges that cross layers – inter-layer edges – than for edges that stay within a single layer – intra-layer edges. Intra-layer edges are used represented as solid lines and the edges that cross layers (i.e., inter-layer edges) as dotted lines.

- A multilayer network is node-aligned (or "fully interconnected") if all the layers contain all nodes. In other words, all nodes are in every layer.
- Coupling edges link the same two nodes (same entity) in different layers;
    - A diagonal multilayer network is layer-coupled if the coupling edges and their weights are independent of the nodes. In other words, for any two layers, the coupling is the same for all nodes (so it depends only on the layers).

b. MULTIPLEX NETWORKS

A complex network (containing non-trivial topological features, different subgraphs – Kim and Wilhelm, 2008) is rarely isolated, and some of its nodes could be part of many graphs, at the same time. Multimodal transportation networks transportation, climatic systems, economic markets, energy-supply networks and the human brain are representative examples of a broad class of real world systems which, rather than being independent, are typically interdependent. In these cases, each network is part of a larger system in which a set of interdependent networks with different structure and function coexist, interact and coevolve. The structural properties of each of these networks and their evolution can depend in a non-trivial way on that of other graphs to which they are interconnected. Consequently, these systems are better represented as multiplexes, i.e. graphs composed by $\boldsymbol{M}$ different layers in which the same set of $\boldsymbol{N}$ nodes can be connected to each other by means of links belonging to $\boldsymbol{M}$ different classes or types (V. Nicosia et al., 2013). Each class of edges corresponds to a unique layer, as well as any node $\boldsymbol{i}$ of the multiplex network consists of $\boldsymbol{M}$ replicas, one for each layer.

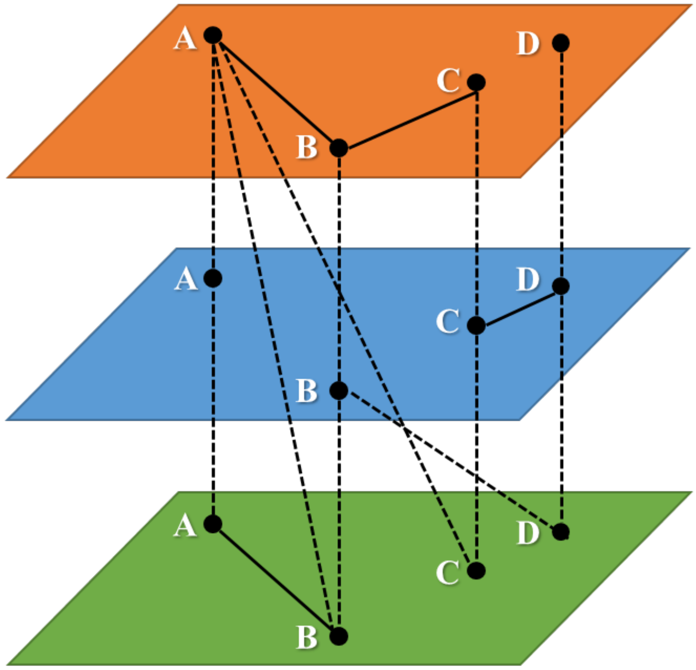


FIGURE 11 – Example of a node-aligned multiplex network (3 layers and 4 nodes) **SOURCE:** Kivelä et al. (2014) pp. 11

Alternatively, one can define multiplex networks as edge-coloured multigraphs, which are networks with multiple types of edges. An edge-coloured multigraph includes the node and the colour sets (which are used for labelling the type of edge). The term colour here defines a label, so edges that are incident to the same node can have the same colour. In this definition of edge-coloured multigraphs, a pair of nodes cannot be adjacent to each other via multiple edges of the same colour once each colour represents a different layer. One can use edge-coloured multigraphs to represent a set of multiple networks that have the same set of nodes in each layer by associating each layer with a unique colour.

c. INTERDEPENDENT VS INTERCONNECTED NETWORKS AND NETWORKS OF NETWORKS

In interdependent network, nodes in two or more monoplex networks are adjacent to each other via edges that are called dependency edges. For example, one can construct an electrical grid and a computer network as a pair of interdependent networks, as the proper function of a router in the computer network can depend on a power station and vice versa. Similarly, interconnected networks, interacting networks, and networks of networks are sets of networks in which some of the nodes from the various networks are adjacent to each other, but the edges that connect different networks do not need to indicate a dependency relation.

If the connections in interdependent networks and similar structures are limited in a certain way, then there is a relationship between them and multiplex networks. In multitype networks and heterogeneous networks, all the nodes are labelled with some "type" and they can be adjacent to nodes that are labelled with either the same or a different type. For example, the nodes in social multitype networks might be labelled with demographic characteristics such as sex, age, or ethnic group.

For interdependent networks and networks of networks (and related frameworks), one needs to map the networks into a flattened graph and then assign colours to nodes according to the subnetwork to which each node belongs.

d. OTHER TYPES OF NETWORKS AND GRAPHS

Other network approaches encountered on literature were for instance, k-partite graphs, networks with both coloured nodes and coloured edges and multilevel networks.

A k-partite network is formally denoted as $G_k = (V_k, E_k)$ where $V_k = \{V_i\}_{i=1}^{k}$ is a collection of k pairwise sets of nodes, such that each set V_i represents nodes of a certain type and E_k is a set of edges, where edges are not allowed between nodes of the same type. Typically, a k-partite graph is a special case of the node-coloured graphs: each node type corresponds to a colour, and the colouring is a proper node-colouring, so two nodes of the same colour cannot be incident to the same edge. Some of the most important works performed were mostly "bipartite" (i.e., 2-partite) networks. Allard et al. (2012) also considered the projection of node-coloured bipartite graphs to several types of multilayer networks. A natural way to map such network to the general multilayer network framework is to consider node colours and edge colours as separate aspects: if two edges have the same colour, then the pair of nodes that are incident to one of these edges must share the same colour combination as the pair of nodes that are incident to the other edge.

Multilevel networks are based on the idea of "multilevel analysis" to networks. In multilevel networks, nodes can have any finite number of types (i.e., "levels") and in which there can be between nodes of the same type or between nodes of "adjacent" types (Wang, et al., 2013). They used the term "macrolevel network" for one level, "micro-level network" for the other level, and "meso-level network" for a network that consists of micro and macro-level nodes, and exclusively inter-level edges.

Multilevel networks fit in a multilayer network framework considering that each level is a layer. The resulting structure amounts to a node-coloured network in which only inter-layer edges between "adjacent" colours (i.e., consecutive layers) are allowed. Two-level networks are equivalent to node-coloured networks with two colours. A type of multilayer network of particular relevance for telecommunication networks (such as the internet) is a "hierarchical multilayer network", in which the bottom layer constitutes a "physical" network and the remaining layers are "virtual layers" that operate on top of the physical layer (Kurant and Thiran, 2006).

e. NODE DEGREE & NEIGHBOURHOOD IN MULTILAYER NETWORKS

The simplest way to generalize the concepts of degree and neighbourhood for multiplex networks is to use network aggregation. Similarly to undirected and unweighted monoplex networks, a node's degree is the number of edges of any type that are incident to a node (i.e., the number of its immediate neighbours). In case of directed networks, the notion of degree can be generalized into in-degree and out-degree, indicating, respectively, the number of incoming and outgoing edges (Nicosia, 2013). One way to understand how central the nodes are, in the context of a multilayer network, is then to project all layers, by summing up the corresponding individual scores of the centrality measures. "For multiplex networks, we can study how the degree is distributed among the different nodes at each layer, but it is also important to evaluate how the degree of a node is distributed across different layers. It is in fact possible that nodes that are hubs in one layer have only few connections, or are even isolated, in another layer. Or, alternatively, nodes which are hubs in one layer are also hubs in the other layers." (Battiston, Nicosia, Latora, 2013, pp. 4).

Alternatively, two nodes can also be considered adjacent if and only if the number of edges that connect them in a multiplex network is larger than some threshold value (Bródka, et al., 2012). This approach defines neighbourhood of a node in a directed multiplex network as the number of different types of edges and taking into account the directions of the edges.

It is possible to define degree and neighbourhood in terms of a focal node and any subset of the layers. The neighbours of a node u as the set of nodes that can be reached by following any edge that starts from node u in any of the layers (Berlingerio et al., 2011).

f. MULTILAYER MEASURES

f.1. COMPRESSION OF LAYERS AND REDUCIBILITY

Many complex systems can be represented as networks consisting of distinct types of interactions, which can be categorized as links belonging to different layers. A fundamental open question is then how many layers are indeed necessary to accurately represent the structure of a multilayered complex system.

An information-theoretical approach is introduced by Domenico et al. (2015) to reduce the dimensionality of the multilayer network, while minimizing information loss. This method aggregates redundant layers and it allows, at the same time, to describe the network using a smaller number of layers and to maximize the distinguishability between the multilayer network and the corresponding aggregated graph.

The method is based on a purely information theoretic perspective, which makes use of the definition of Von Neumann entropy of a graph. The figure below illustrates the overall process of a layer aggregation and reduction of the multilayer network dimension.

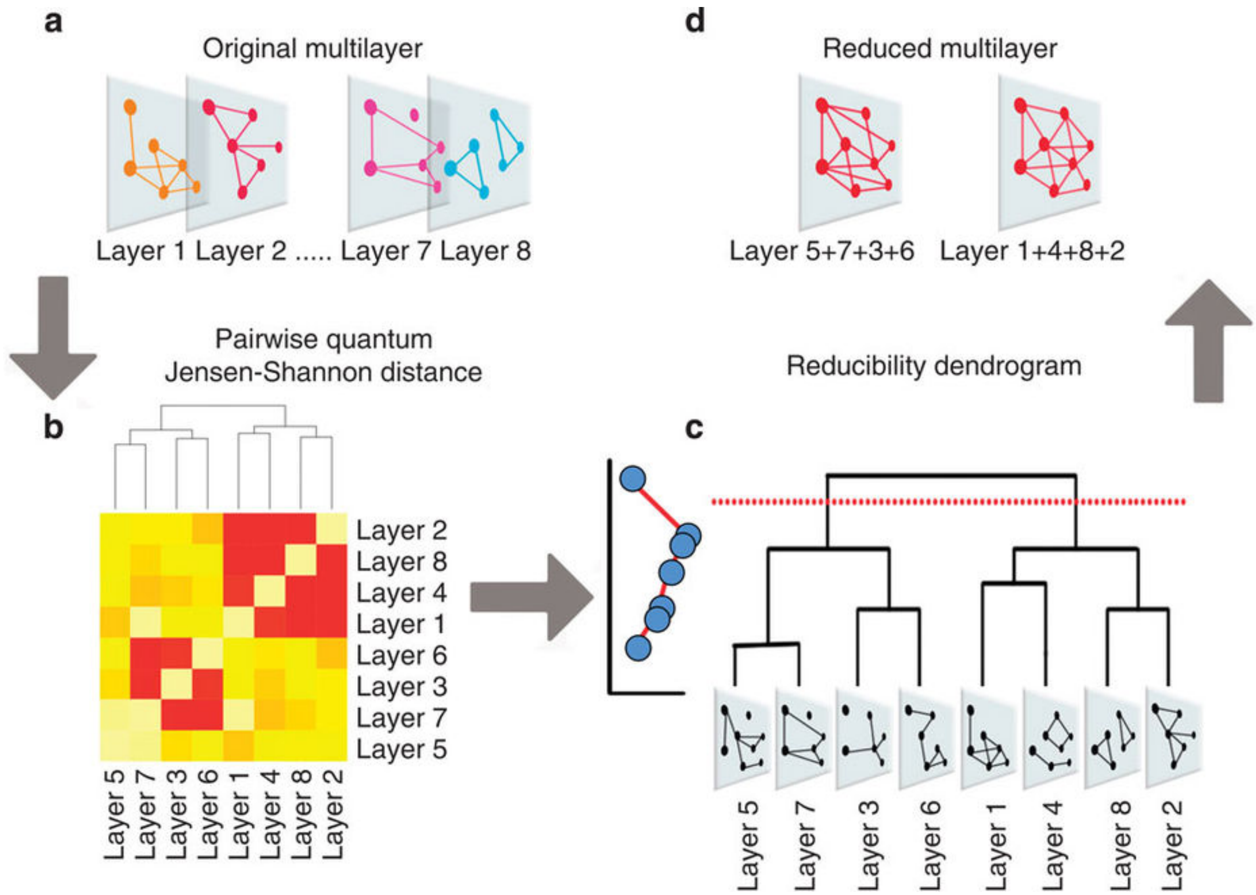


FIGURE 12 – Layer aggregation and structural reducibility of multilayer networks **SOURCE:** Domenico et al. (2015)

Given a multilayer network (**a**), the metric distance is computed between each pair of its layers (**b**) by means of the Jensen-Shannon divergence, which is a proxy for layer redundancy. Such resulting distance matrix allows to perform a hierarchical clustering (according to the ward algorithm), whose output is a hierarchical diagram (a dendrogram) whose leaves represent the initial layers and internal nodes denote layer merging (**c**). At each step, the two clustered layers (or group of layers) corresponding to the smallest value of the Jensen-Shannon matrix are aggregated and the quality of the new layer configuration in terms of distinguishability from the aggregated graph is quantified by a global quality function, shown by the curve on the left-hand side of (**c**). The best partition is the one for which the global quality function is maximal (**d**).

f.2. ASSORTIVITY

The assortativity is a measure of similarity between layers. It computes a preference for a network's nodes to attach to others that are similar in some way (Domenico et al., 2015). Several indicators are proposed in the literature:

- **Mean global node overlapping:** it measures the fraction of node which are common (i.e., non-isolated) to all players.
- **Inter-layer Assortativity (Pearson**): it calculates the Pearson correlation between the degree (strength) of nodes and their couterparts in other layers, for all pairs of layers.
- **Inter-layer Assortativity (Spearman):** it computes the Spearman correlation between the degree (strength) of nodes and their couterparts in other layers, for all pairs of layers.

g. COMMUNITY DETECTION

One can examine the global organization of nodes into modules (i.e. 'communities') through an algorithmic calculation of community structure (Porter et al., 2009). To do this, one takes into account both intralayer and interlayer edges, and one seeks densely connected sets of nodes (i.e. communities) that are sparsely connected to each other when compared with some multilayer random-graph (null) model (Kivelä et al., 2014).

h. NETWORK FEATURES

A final scheme with an overview of the different features for describing multiplex network structures is introduced. Each feature can be widely used in many areas.

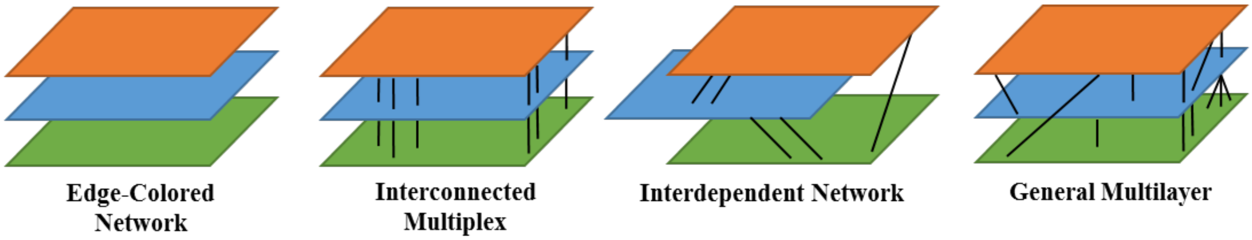


FIGURE 13 – Network Features **SOURCE:** Muxviz

2.4. Author Synthesis

Theme	Author	Publication	Year
Monoplex Network	Adams, Moody and Morris	Sex, drugs, and race: How behaviours differentially contribute to the sexually transmitted infection risk network structure	2013
	Ashwin and Field	Heteroclinic networks in coupled cell systems	1999
	Barabási and Albert	Emergence of scaling in random networks	1999
Network of Networks	Bianconi and Dorogovtsev	Multiple percolation transitions in a configuration model of network of networks	2014
	Bianconi, Dorogovtsev and Mendes	Mutually connected component of network of networks	2014
Complex Network	Cowan, Chastain, Vilhena, Freudenberg and Bergstrom	Nodal dynamics, not degree distributions, determine the structural controllability of complex networks	2012
	Albert, Jeong and Barabási	Error and attack tolerance of complex networks	2000
	Arenas, Duch, Fernández and Gómez	Size reduction of complex networks preserving modularity	2007
	Barrat, Barthelemy and Vespignani	Dynamical Processes on Complex Networks	2008
	Boccaletti, Latora, Moreno, Chavez and Hwang	Complex networks: Structure and Dynamics	2006
	Bródka, Musiał and Kazienko	A method for group extraction in complex social networks	2010
	Barrat, Barthelemy, Pastor-Satorras, and Vespignani	The architecture of complex weighted networks	2004
Interdependent Network	Baxter, Dorogovtsev, Goltsev and Mendes	Avalanche Collapse of Interdependent Networks	2012
	Berezin, Bashan and Havlin	Comment on “percolation transitions are not always sharpened by making networks interdependent”	2013
	Brummitt, D’Souza and Leicht	Suppressing cascades of load in interdependent networks	2012

THEME	AUTHOR	PUBLICATION	YEAR
	Buldyrev, Parshani, Paul, Stanley and Havlin	Catastrophic cascade of failures in interdependent networks	2010
	Buldyrev, Shere and Cwilich	Interdependent networks with identical degrees of mutually dependent nodes	2011
MULTIDIMENSIONAL NETWORK	Berlingerio, Coscia and Giannotti	Finding redundant and complementary communities in multidimensional networks	2011
	Berlingerio, Coscia, Giannotti, Monreale and Pedreschi	Foundations of multidimensional network analysis	2011
	Berlingerio, Coscia, Giannotti, Monreale and Pedreschi	The pursuit of hubbiness: Analysis of hubs in large multidimensional networks	2011
	Berlingerio, Coscia, Giannotti, Monreale and Pedreschi	Multidimensional networks: Foundations of structural analysis	2013
	Berlingerio, Pinelli and Calabrese	ABACUS: frequent pattern mining-based Community discovery in multidimensional networks	2013
	Barrett, Henzi and Lusseau	Taking sociality seriously: The structure of multi-dimensional social networks as a source of information for individuals	2012
MULTILAYER NETWORK	Bródka, Kazienko, Musiał and Skibicki	Analysis of neighbourhoods in multi-layered dynamic social networks	2012
	Bródka, Skibicki, Kazienko and Musiał	A degree centrality in multi-layered social network	2011
	Bródka, Stawiak and Kazienko	Shortest path discovery in the multi-layered social network	2011
	Cardillo, Zanin, Gómez-Gardeñes, Romance, García del Amo and Boccaletti	Modelling the multi-layer nature of the European air transport network: Resilience and passengers re-scheduling under random failures	2013
MULTIPLEX NETWORK	Bargigli, Lasio, Infante, Lillo and Pierobon	The multiplex structure of interbank networks	2013
	Battiston, Nicosia and Latora	Metrics for the analysis of multiplex networks	2013
	Baxter, Dorogovtsev, Mendes and Cellai	Weak percolation on multiplex networks	2013

Theme	**Author**	**Publication**	**Year**
	Bianconi	Statistical mechanics of multiplex networks: Entropy and overlap	2013
	Bonacina, D'Errico, Moretto, Stefani and Torriero	A multiple network approach to corporate governance	2014
	Brummitt, Lee and Goh	Multiplexity-facilitated cascades in networks	2012
	Buono, Zuzek, Macri and Braunstein	Epidemics in partially overlapped multiplex networks	2013
	Cardillo, Gómez-Gardeñes, Zanin, Romance, Papo, del Pozo and Boccaletti	Emergence of network features from multiplexity	2013
	Cellai, López, Zhou, Gleeson and Bianconi	Percolation in multiplex networks with overlap	2013
	Corominas-Murtra, Fuchs and Thurner	Detection of the elite structure in a virtual multiplex social system by means of a generalized k-core	2013
	Allard, Noël, Dubé and Pourbohloul	Heterogeneous bond percolation on multitype networks with an application to epidemic dynamics	2009

Table 3 – Author Synthesis **Source:** Kivelä et al. (2014) pp. 41-5

CHAPTER 3 – DATA AND MODEL IMPLEMENTATION

This section mainly focuses on the implementation of the proposed methodology. Several analyses are carried out, including the mono and multiplex network case studies.

This chapter also describes the overview of the steps and methods followed in collecting and processing the raw data (real world datasets).

3.1. DATA COLLECTION

As our study aims for investigating the Portuguese urban system and applying analytical domains to a consistent European database, this research includes different datasets for each category analysis. Those datasets provide different but equally important insights on the global Portuguese mobility and trade and on the European commercial trade, investment and migration indicators. Some data (at European level) is accessible on the internet, through free available data sets, given that data have become widely available. Considering the theme in geographical and relevancy terms, information provided by several governmental institutions, both national and international ones, has allowed to extend this research to a more realistic, consistent and valid one. Nevertheless, each dataset was restructured and redesigned as data matrix where the indices from left to right determine the direction of the interaction. Databases' sources and references are presented below: databases dimension is 308 by 308 for the Portuguese case study – each entity corresponds to a Portuguese municipality –, and 28 by 28 for the European Union research – each individual corresponds to a member of EU; the union reached its current size of 28 member countries with the accession of Croatia on July 1st, 2013.

a. PORTUGUESE URBAN SYSTEM DATASETS

For the Portuguese urban system analysis, two different datasets are collected:

1. Commuting interactions due to professional reasons in the territorial unit of the employed resident population by place of residence or destination (INE, 2011);

2. Commuting interactions due to academic reasons in the territorial unit of the student resident population attending higher education by place of residence or destination (INE, 2011).

a.1. Commuting Interactions due to Professional Reasons Dataset

The commuting mobility of employed population dataset is sorted out alphabetically by place of residence, grouping data by municipality (it was originally organized by civil parishes). A total of 4.030.448 commutes occurred in 2011 between municipalities for work purposes, according to Census 2011 data. Among them, 1.327.733 correspond to commutes between different municipalities. This means that 2/3 of the Portuguese employed population was working in the same municipality as their place of residence. Each horizontal line from the data matrix corresponds to the starting point; each column represents the place of departure.

The following table represents the head of this dataset (5×5). For instance, pair $(Albergaria\ a\ Velha, Águeda)$ represents that in 2011, 900 people travel from Albergaria-a-Velha to Águeda due to work reasons.

	Abrantes	Águeda	Aguiar da Beira	Alandroal	Albergaria-a-Velha
Abrantes	15805	0	0	0	1
Águeda	4	23119	0	0	586
Aguiar da Beira	0	0	1881	0	0
Alandroal	2	0	0	1811	0
Albergaria-a-Velha	1	900	1	0	10157

Table 4 – PT Urban System - Head of commuting due to professional reasons dataset **Source:** INE, 2011

a.2. Commuting Interactions due to Academic Reasons Dataset

The commuting mobility of resident students attending higher education dataset was grouped by municipality (it was originally organized by civil parishes). Data refers to the year 2011; each row of the obtained adjacency matrix corresponds to the place of residence and each column denotes the place of departure. Each index (x, y) of the dataset shall be intended as the number of population attending higher education from the municipality x to the municipality y. The table below illustrates the head of this dataset (5×5).

	ABRANTES	**ÁGUEDA**	**AGUIAR DA BEIRA**	**ALANDROAL**	**ALBERGARIA-A-VELHA**
Abrantes	5297	0	0	0	0
Águeda	1	6968	0	0	145
Aguiar da Beira	0	0	662	0	0
Alandroal	0	0	0	596	0
Albergaria-a-Velha	1	120	1	0	3739

TABLE 5 – PT Urban System - Head of commuting due to academic reasons dataset **SOURCE:** INE, 2011

A total of 1.890.083 commutes were conducted in 2011 due academic reasons. Among them, only 292.473 (around 15%) were performed between different municipalities.

b. EUROPEAN UNION DOMAINS DATASETS

For the EU domains analysis, four different datasets are collected:

1. Trade transactions in terms of imports in the European Union territorial unit between EU countries by point of exit into the importing country, thousands of US $ (WITS, 2011);
2. Trade transactions in terms of exports in the European Union territorial unit between EU countries by point of exit into the exporting country, thousands of US $ (WITS, 2011);
3. Net inflows of investment in the European Union territorial unit between EU countries by place of investment, millions of US $ (World Data Bank, 2011);
4. Bilateral remittance estimates using Migrant Stocks, Host Country Incomes, and Origin Country Incomes, millions of US $ (World Data Bank, 2011).

b.1. TRADE TRANSACTIONS IN TERMS OF IMPORTS DATASET

An advanced query in Trade Data section of World Integrated Trade Solution (WITS8) web application was the source of database representing import trade values between the 28 EU countries for the 2011 reference year, filtering all products and services[9].

Dataset corresponds to a matrix 28×28. Each line from the adjacency matrix, obtained from the initial dataset, corresponds to the starting point – country member of EU that performed import; each column represents the EU country from which the import was made. Each pair (x, y) provides the import value (in current U.S. dollars, thousands) of all traded products and services between the reporting country x and its country partner y. The table below depicts the head of this adjacency matrix (6×6) in both matrix and edgelist formats. For instance, pair (AUT, BEL) represents that in 2011, around 2.698.682 thousand $ were imported by Austria from Belgium. Similar conclusions can be made for the pair $(1, 2, 2698682)$. The original dataset included, for some countries, non-empty values for the pairs (x, x) (identical pairs). We found it, for instance the pair $(POR, POR) = (13670, 13670)$. Despite that, in this context self-loops were ignored. For programming reasons, it was needed to structure data in an edgelist format. More than a simple tie linking each countries pair, we are interested in analysing the richness of network by considering different trade values – we are in the presence of a weighted one-mode network. This matrix format has 3 columns: the first column represents the id of the reporting country; the second is the id of the partner country and the third identifies the weight of the tie.

	AUT	BEL	BGR	CYP	CZE	DEU
AUT	0	2698682	530586	0	6272226	62559523
BEL	2660443	0	1098504	0	3890898	65926343
BGR	1000050	519478	0	108508	536375	3285984
CYP	53774	171480	55995	0	37328	688178
CZE	4478942	2620154	260143	0	0	37466357
DEU	43472260	44304408	2708242	0	40965704	0

[8] **WITS** is an integrated platform allowing users to access and retrieve information on trade and tariffs. This tool is an outcome of the World Bank Trade Competitiveness Diagnostic toolkit, developed by the International Trade Unit, in collaboration with the UNCTAD, ITC, UNSD and WTO. Available at: https://wits.worldbank.org/

[9] food and live animals, beverages and tobacco, crude materials, inedible, except fuels, mineral fuels, lubricants and related materials, animal and vegetable oils, fats and waxes, chemicals and related products, manufactured goods classified chiefly by material, machinery and transport equipment, miscellaneous manufactured articles and other commodities and transactions

REPORTING_ID	**PARTNER_ID**	
1	2	2698682
1	3	530586
1	5	6272226
1	6	62559523
1	7	720168

TABLE 6 – EU Domains - Head of Imports dataset (in different formats) **SOURCE:** WITS (2016)

This matrix is an example of a fully connected or complete network: all nodes are interconnected. In a directed network, the initial number of connections c grows in a quadratic proportion with the number of nodes n: $c = n(n-1) = 28(28-1) = 756$ (ignoring self-loops). For simplification purposes, for each horizontal line, it was computed the first quartile in order to ignore edges under that value. In other words, for each reporting country, the highest 20 import trade values (out of the initial 28) were considered for the analysis of the network. The lowest 8 import trades values were then ignored.

b.2. TRADE TRANSACTIONS IN TERMS OF EXPORTS DATASET

Similar to the previous import data collection, the international exporting trades between the 28 *EU* countries for the year 2011 was collected after the same advanced query in *WITS*. All products and services were selected as well and values are expressed in current U.S. dollars, thousands.

In this specific case, each horizontal row of the obtained adjacency matrix corresponds to the origin country from which products and services were exported and each vertical column represents the exporting recipient country. For instance, the pair of countries (AUT, BGR) of the following table and $(1, 3, 795019)$ of the following edgelist table represents that during 2011, Austria exported to Bulgaria 795019 thousand of U.S. dollars considering the whole range of products and services.

	AUT	BEL	BGR	CYP	CZE	DEU
AUT	0	2118377	795019	0	5541754	48212642
BEL	4323491	0	0	0	3698126	78822304
BGR	506867	1383980	0	78442	280581	3209537
CYP	7503	15958	9982	0	13055	88293
CZE	6866589	3847731	561178	0	0	49622901
DEU	66677454	59994764	0	0	36767837	0

REPORTING_ID	PARTNER_ID	
1	2	2118377
1	3	795019
1	5	5541754
1	6	48212642
1	7	792463

TABLE 7 – EU Domains - Head of Exports dataset (in different formats) **SOURCE:** WITS, 2016

In the initial dataset, some self-loops were also found and ignored, due the main purpose of this research. To obtain the most relevant findings from the interconnected graph, similarly, only the 20 highest export trade values were considered for each exporting country.

b.3. FDI FINANCIAL FLOWS DATASET

Foreign direct investment data was collected from *OECD.Stat*, *OECD* International direct investment database (OECD, 2016), in current U.S. dollars, millions. This database refers to the year 2011 and it was a result of a compilation in respect to the host or investing country. The data matrix structure is similar to the previous selected variables: an adjacency matrix with dimension 28×28 representing the investment flows between each reporting country, stated in each line of the table, and its partner country, mentioned in the respective column.

Due some the lack of information, it was not possible to have available data for all the 28 *EU* countries in analysis: FDI financial flows were initially based on statistics provided by the *OECD* member countries and not by the *EU* member states. In practice, Bulgaria, Croatia, Cyprus, Latvia, Lithuania, Malta, and Romania, current *EU* member states but not *OECD* member countries, had not available data. This implies that for this variable, a 0 value can mean both absolute zero investments observed between the considered countries and missing value.

Direct investment includes not only equity investment capital but also reinvested earnings and other non-equity capital transactions (mainly intra-company loans). FDI flows can have a negative sign (reverse flows) if at least one of the components in the above definition is negative and not offset by positive amounts of the remaining components.

The following table shows the head of the first dataset (6×6) in both matrix and edgelist formats. For instance, pair (AUT, CYP) represents that in 2011, Austria had imported from Cyprus around 39 thousand USD. Same conclusions can be made for the pair $(1, 4, 39)$ in the subsequent table.

	AUT	**BEL**	**BGR**	**CYP**	**CZE**	**DEU**
AUT	0	0	467	39	506	4850
BEL	0	0	0	0	625	5954
BGR	0	0	0	0	0	0
CYP	0	0	0	0	0	0
CZE	34	21	0	0	0	0
DEU	9816	8212	0	175	2106	0

REPORTING_ID	**PARTNER_ID**	
1	3	467
1	4	39
1	5	506
1	6	4850
1	7	43

TABLE 8 – EU Domains - Head of FDI financial flows dataset (in different formats) **SOURCE:** OECD.Stat, 2016

b.4. BILATERAL REMITTANCES DATASET

Bilateral remittances matrix was an estimation based on the methodology developed by Ratha and Shaw (2007)[10], sourced in *World Bank*. The remittance date reference is 2011, disaggregated using host country and origin country incomes. Data matrix was reduced to a 28×28 dimension,

[10] "South-South Migration and Remittances", Development Prospects Group, World Bank
Available at: worldbank.org/prospects/migrationandremittances

filtering countries by the ones belonging to the European Union. Reasonably, no values were found in the matrix' diagonal: remittance flows are the result of people living outside of their countries of birth. The following table shows the head of the remittance adjacency matrix (6×6). Each pair (x, y) shall be read as the total remittance inflows estimate for the year 2011, from country x to country y (x is the giver while and y is the receiver). Same conclusions can be made for the pair $(1, 2, 2698682)$.

This dataset describes the remittance sent by immigrant people from their current country to the birth country in 2011. Values are expressed in current U.S. dollars, millions.

	AUT	**BEL**	**CZE**	**DNK**	**EST**	**FIN**
AUT	0	48	12	96	0	254
BEL	22	0	4	0	0	16
CZE	0	0	0	0	0	0
DNK	9	0	0	0	0	0
EST	2	6	7	0	0	3
FIN	39	18	6	3	0	0

REPORTING_ID	**PARTNER_ID**	
1	2	48
1	3	12
1	5	254
1	7	7
1	9	1

TABLE 9 – EU Domains - Head of Bilateral Remittance dataset (in different formats) **SOURCE:** World Data Bank, 2016

3.2. MONOPLEX NETWORK ANALYSIS

a. PORTUGUESE URBAN SYSTEM ANALYSIS

a.1. COMMUTING INTERACTIONS DUE TO PROFESSIONAL REASONS

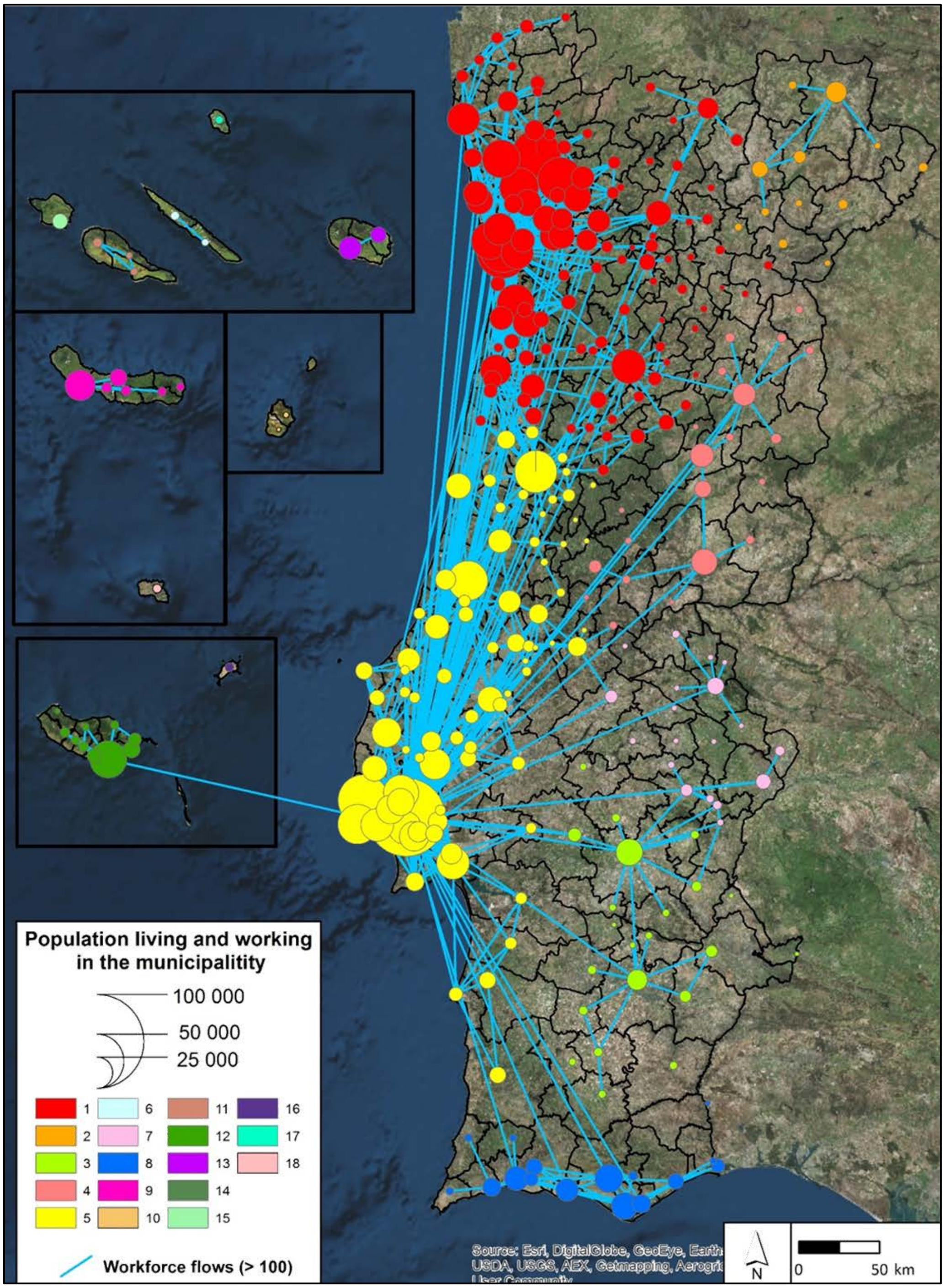


FIGURE 14 – PT Urban System - Commuting due to professional reasons: Community network *SOURCE***:** Author, ArcGIS

The graph above shows the commuting mobility of employed population in Portugal by municipality in 2011. This variable includes both employed resident population outside the territorial unit and employed non-resident population in the territorial unit. For the sake of readability, the graph representation omits intra-layer edges with weights lower than 100. Each vertex represents a municipality; each edge represents a commuting link due to professional reasons. Edges are organized by colour, using the concept of community detection: municipalities with the same colour are densely connected subgraphs (short random walks tend to stay in the same community). Due the high number of municipalities, the researcher chooses to set a parameter of 50 (approximately 15% of the total number of municipalities) as the maximum length of the random walks/iterations to perform from one node to another.

Community detection used in the previous graph, under the clustering walktrap function, grouped municipalities into **18** different clusters.

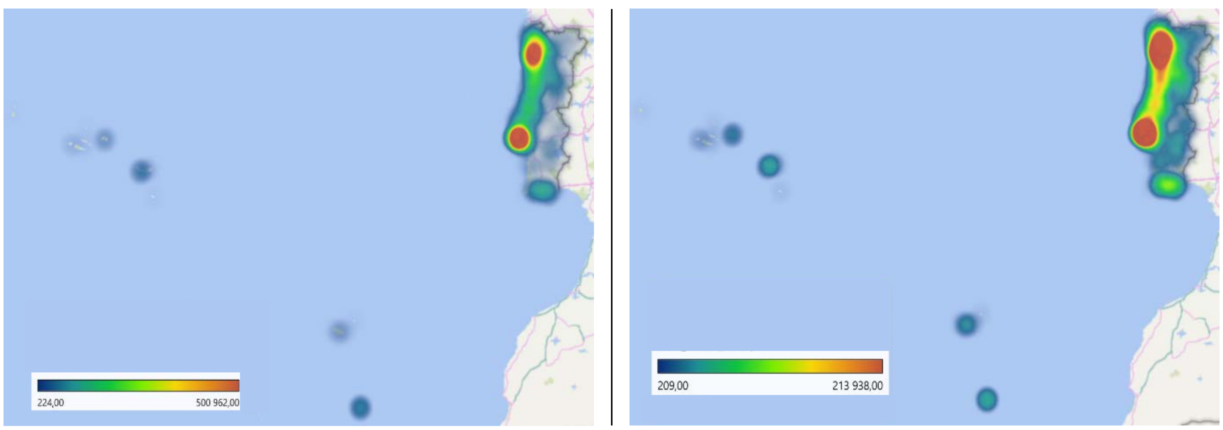


FIGURE 15 – PT Urban System - Commuting due to professional reasons: In / out degree **SOURCE:** Author, Excel

A more detailed analysis of **FIGURE 13** and **FIGURE 14** identifies two geographic poles surrounding the two biggest large cities of Portugal: Porto and Lisbon, both in and outgoing commuting due professional reasons. The contiguous areas expressed in **FIGURE 13** clearly describe the two most dynamic areas of Portugal, with more intense suburbanization processes. These two cities, driving forces of the Portuguese economic development and urbanization, seem to diffuse to their peripheral suburban areas, spreading in the case of Lisbon to cities like Sintra, Almada, Cascais, Oeiras and Setúbal, and to Maia, Matosinhos, Aveiro and Braga for the suburbs of Porto. These cities present as well, among all municipalities, high values for node degree, and in and out degree centrality measures, confirming their importance as secondary assistant cities to the two metropolises.

In Algarve, it is evident a smaller pole of attraction when considering commuting due work reasons; the same applies to the archipelagos of Madeira (Funchal) and the Azores (particularly the island of Sao Miguel, Ponta Delgada).

From a macro point of view, this variable mainly reflects the disparities between the two metropolitan areas of Portugal, as well as between the coast and the interior of the continent. Depopulation and the small attractiveness in terms of work is reflected mostly when analysing the municipalities where employed population are originally from (In-degree map). Looking to the Municipalities receiving employment force from other Municipalities, some other small poles seemed to be highlighted; such small urban areas include Leiria, Viana do Castelo, Santarem, Vila Real and Covilhã municipalities.

Observing more into detail community detected in **FIGURE 13**, to a certain extent, the obtained clustering mostly reflects the NUTS III representation of Portugal: first cluster includes most of the littoral north municipalities, second module is formed by interior north of Portugal municipalities, third group includes south-centre interior municipalities, followed by centre interior of Portugal. Algarve then forms a cluster, followed by some municipality aggregations from archipelagos of Madeira and Azores.

Other centrality measures are also crucial in explaining different behaviours of municipalities. When analysing betweenness, looking only to the ties connections (alfa=0), Lisbon, Coimbra and Porto are, among all municipalities, the ones highly emerging on paths between other municipalities. Other district municipalities such as Viseu, Évora, Aveiro or Braga follow as the second most important set of municipalities showing up in paths between other municipalities. This metric slightly replicates Portugal division territory in districts. If only the volume of employed people is considered, similar conclusions can be made. In conclusion, metropolis centres of Portugal, followed by district division are both reflected in terms of connections and volume. Closeness centrality, considering only ties connections, shows that in 2011 some important cities within two metropolises are important central nodes as they seem to have the smaller distances for all other nodes. Such conclusions are reflected, for instance, in Sintra, Vila

Nova de Gaia, Santa Maria da Feira and Loures. The worst connected nodes in terms of closeness are Corvo, Lagoa and Lajes das Flores (Azores). Considering closeness only in terms of commuting numbers, as the network is highly connected, no precise conclusions could be made.

Taking into consideration outcome provided by *R* for reciprocity measure, if a given municipality A is connected to municipality B due to professional reasons, then there is about 60% chance that also municipality B connects back municipality A due the same reason.

Along with work, studies also are an important reason for commuting in a society.

a.2. COMMUTING INTERACTIONS DUE TO ACADEMIC REASONS

The graph below represents commuting mobility of population attending higher education in Portugal by municipality in 2011. Data collected includes resident population attending higher education outside the territorial unit and non-resident population attending higher education in the territorial unit. Each vertex represents a municipality and each edge represents a commuting link due to academic reasons. Edges are organized by colour, using the clustering walktrap function: municipalities with the same colour are densely connected subgraphs (short random walks tend to stay in the same community). Due the high number of municipalities, the researcher chooses to set a parameter of 140 (approximately 45% of the total number of municipalities) as the maximum length of the random walks/iterations to perform from one node to another. For the sake of readability, this paper omits intra-layer edges with weights lower than 100.

The territorial network below clustered municipalities into **8** different modules.

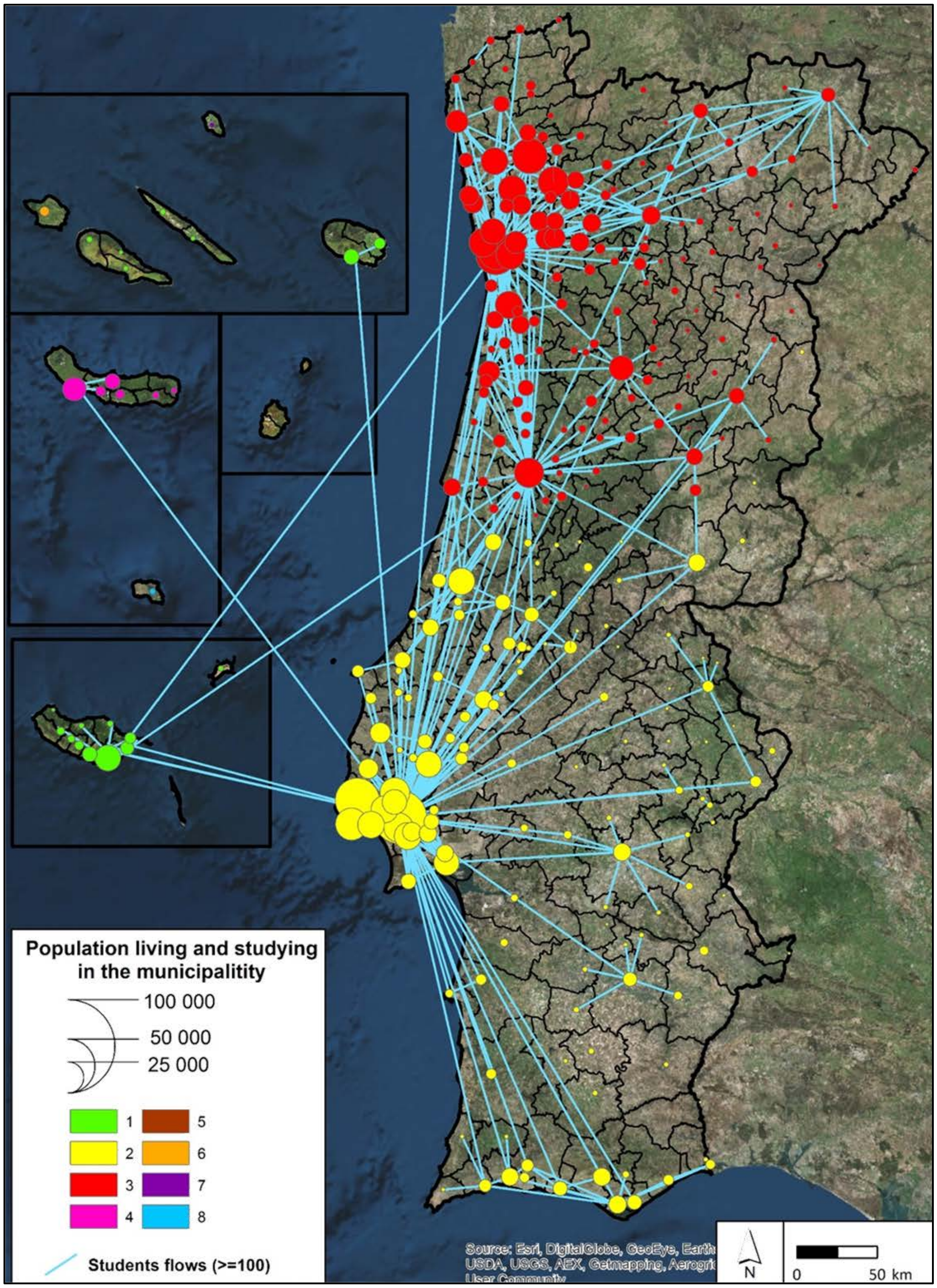


FIGURE 16 – PT Urban System - Commuting due to academic reasons: Community network ***SOURCE*:** Author, ArcGIS

Commuting interactions due to academic reasons determine, by analysing **FIGURE 15**, two major driving forces across Portugal territory: Lisbon and Porto emerge as the main knowledge institutional resources with greater national visibility. Essentially, the most significant commutes are made to and from the metropolis and their suburbs. A deepened understanding allows also to emphasize other regional clusters.

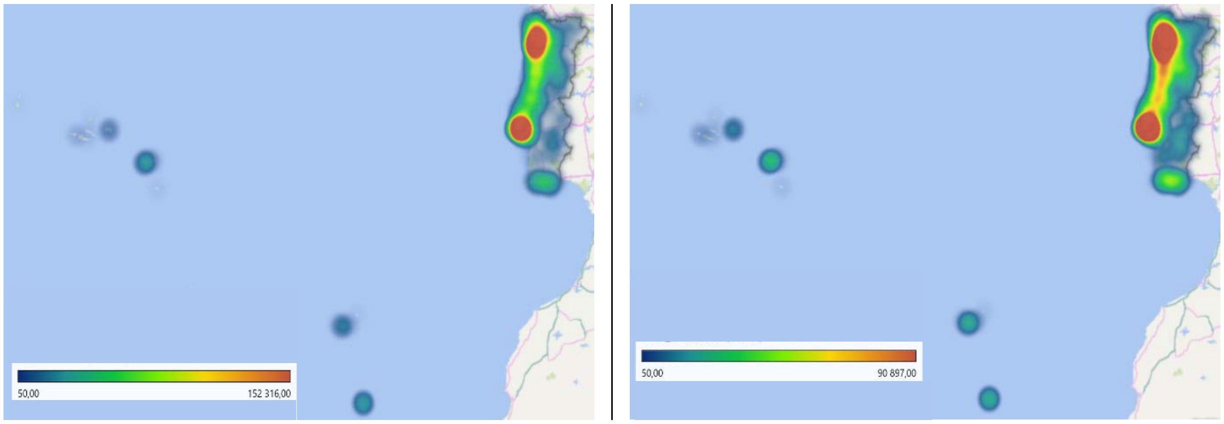


FIGURE 17 – PT Urban System - Commuting due to academic reasons: In / out degree map ***SOURCE*:** Author, Excel

Taking also into consideration betweenness and closeness outcomes (**TABLE 23**), Coimbra, Vila Nova de Gaia, Aveiro, Braga and Vila Real are municipalities with, in 2011, good performance both in terms of ties/weights (links connecting different municipalities/volume of employed residents commuting). Indeed, these municipalities incorporate most of the well-ranked Portuguese universities according to Times Higher Education[11], one of the most worldwide prestigious ranking: University of Aveiro, Coimbra, Lisbon and Porto were placed in 401-500 ranking; University of Minho and Nova University of Lisbon were positioned in 501-600; University of Beira Interior and ISCTE-University Institute of Lisbon were ranked in 601-800. Closeness centrality outcomes, in particular, remark a wide range of suburb municipalities of Porto (e.g.: Vila Nova de Gaia, Porto, Maia, Valongo, Gondomar and Matosinhos) with lower total distance from all other municipalities, in other words, the most central nodes of the network.

Regarding the south region of Portugal and archipelagos of Madeira and Azores, despite that in the general Portuguese panoramic these areas have low education influence, they seemed to have an important role in such sub regions, namely Faro, Loulé and Portimão in Algarve region, Funchal in Madeira, and Angra do Heroísmo and Ribeira Grande in Azores.

[11] Available at https://www.timeshighereducation.com/world-university-rankings

Furthermore, **FIGURE 15** nearly reflects the main discrepancies between the North and South of Portugal. The first cluster includes most of the municipalities placed in the North 7 centre of Portugal, the second module has municipalities from Lisbon and its suburbs, Alentejo and Algarve (Faro district) regions; third to eight groups correspond to different municipalities of Azores and Madeira islands.

Reciprocity measure for commuting interactions due to academic reasons was in 2001, by using *R*, 41,4%. Therefore, there is around 41,4% possibility that if municipality A links to municipality B, then B also links back to A.

In conclusion, results obtained in commuting interactions due to academic reasons were slightly similar to the ones obtained for commuting interactions due to professional reasons, mainly showing the supremacy and importance of Lisbon and Porto regions to the overall development of Portugal.

b. EUROPEAN UNION DOMAINS ANALYSIS

b.1. TRADE TRANSACTIONS IN TERMS OF IMPORTS

The following figure describes the network obtained for the international merchandise and services trade relations in terms of imports between each pair of countries belonging to the European Union 28 for the year 2011. An import is a good or service brought into one country from another. Each edge represents a trade link between countries; each vertex represents a EU-28 country. Edges are organized by colour, using the walktrap algorithm for community detection: countries with the same colour are densely connected subgraphs (short random walks tend to stay in the same community).

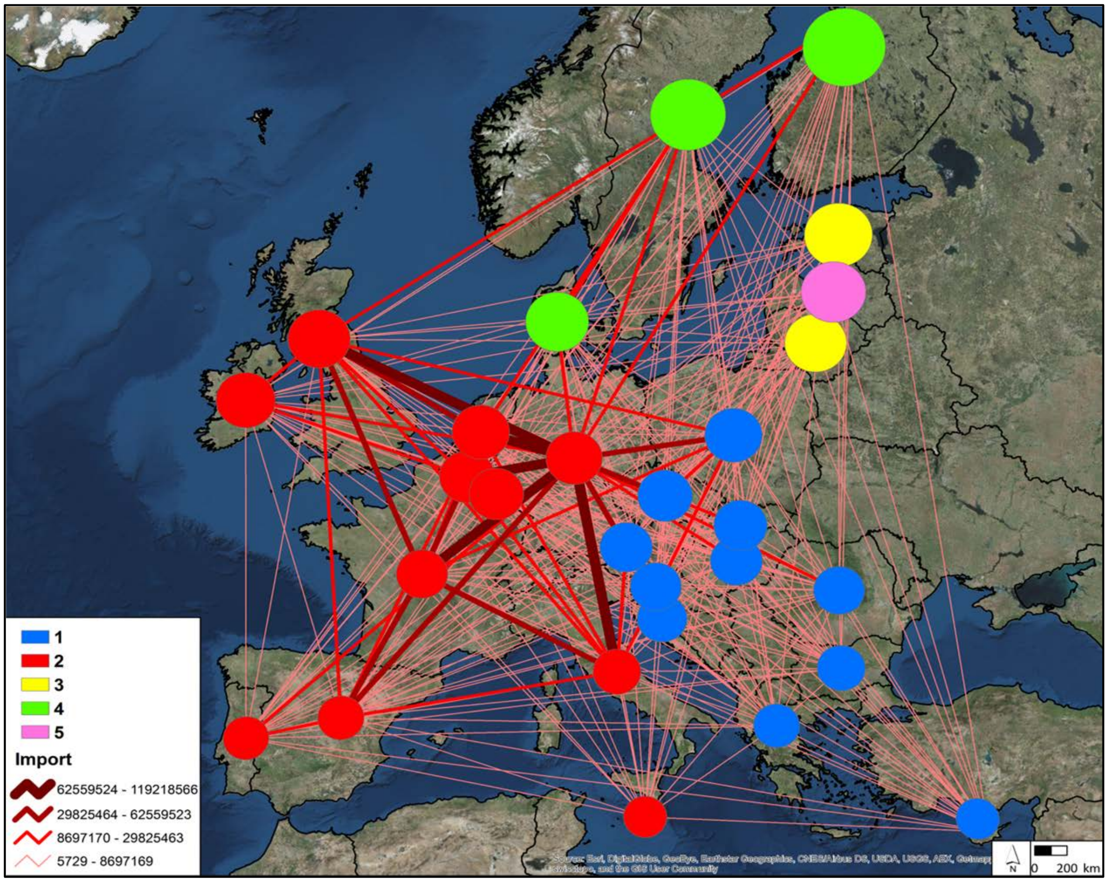


FIGURE 18 – EU Domains – Imports: Community network **SOURCE:** Author, ArcGIS

The suggested community detection in the previous territorial network, under the clustering walktrap function, divides countries into 5 groups:

[1] Austria, Bulgaria, Cyprus, Czech Republic, Greece, Croatia, Hungary, Poland, Romania, Slovakia and Slovenia;

[2] Belgium, Germany, Spain, France, United Kingdom, Ireland, Italy, Luxembourg, Malta, Netherlands and Portugal;

[3] Estonia and Lithuania;

[4] Denmark, Finland and Sweden;

[5] Latvia.

CENTRALITY MEASURES

At this stage, the analysis focused by computing main basic graph indicators (centrality measures), which indicate the most important vertices within a graph.

Under the imports analysis, the researcher is interested to understand which countries import the most. In empirical terms, we want to know which are the countries with more outgoing links, meaning we must compute the out-degree. Nevertheless, for the purpose of analysis, the researcher also considers HDI (Human Development Index) – a summary measure, the ultimate criteria for assessing (not economic growth alone) the main key dimensions of human development: a long and healthy life, being knowledgeable and have a decent standard of living (**TABLE 24**). Globally, this indicator led us conclude that countries better ranked in HDI tend to be the ones with higher level of imports. Indeed, one country, due its dimension (in terms of both population and market needs), has advantages buying from other countries: one country may be more productive than others.

Others important considered measures are the betweenness and the closeness. By using *R*, **TABLE 25** shows, per each EU country, the different values of betweenness and closeness, according to the different sets of parameter alpha. Regarding betweenness outcomes, Germany stood out as the member state with the highest imports trade in terms of volume when compared with the other EU countries (it has the highest value of betweenness – 634, for alpha=1). However, in terms of trade connections (alpha = 0), countries like Croatia, Denmark Sweden, Finland and Italy, despite having a significant lower volume of imports between other countries, they buy from several different EU sources. Generalizing, on the one hand, in 2011, Germany imported the highest amount in terms of volume; on the other hand, some northern and southern Europe countries bought from multiple EU countries sources but in small magnitudes. When analysing closeness outcomes, it is relatively identical to cross from a given country to another. No country was highlighted when trying to reach other(s) easily. This is due the high level of ties between them. The parameter in closeness measure did not have a significant impact as in betweenness outcomes.

Along with imports, exports form the backbone of international trade.

b.2. TRADE TRANSACTIONS IN TERMS OF EXPORTS

The higher the value of exports entering a country, compared to the value of imports, the more positive that country's balance of trade becomes. An export is a function of international trade whereby goods produced in one country are shipped to another country for future sale or trade.

The figure below shows the obtained network for the EU-28 in terms trade exports in 2011. Each link represents a trade between countries; each vertex denotes a country. Community detection method was similarly used to organize edges by colour (countries with the same colour are densely connected subgraphs – short random walks tend to stay in the same community).

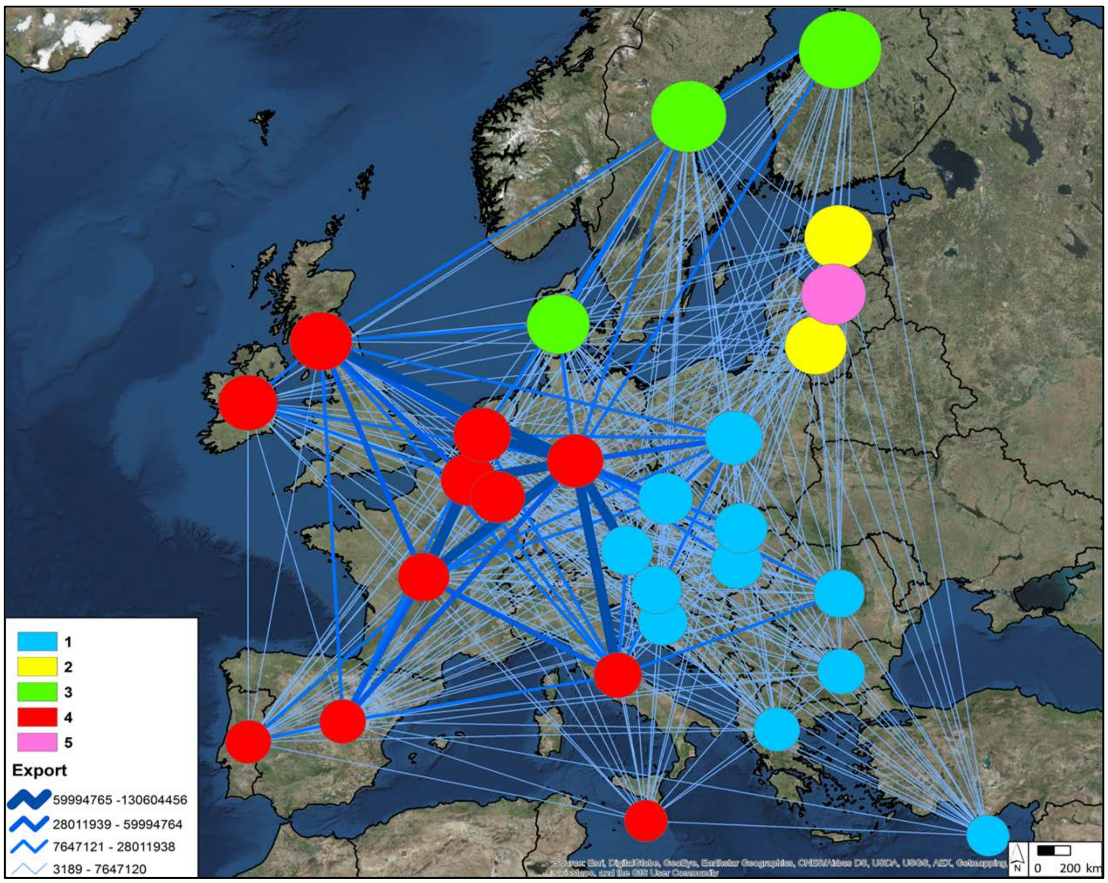


FIGURE 19 – EU Domains – Exports: Community network **SOURCE:** Author, ArcGIS

Countries were grouped in 5 different communities, under the under the clustering walktrap function:

[1] Austria, Bulgaria, Cyprus, Czech Republic, Greece, Croatia, Hungary, Poland, Romania, Slovakia and Slovenia;

[2] Estonia and Lithuania;

[3] Denmark, Finland and Sweden;

[4] Belgium, Germany, Spain, France, United Kingdom, Ireland, Italy, Luxembourg, Malta, Netherlands and Portugal;

[5] Latvia.

This clustering method mainly grouped countries by region. Primarily, most of the East Europe countries become in the same module, then Central and Southern European countries formed another cluster; then Northern European countries created a cluster, Estonia and Lithuania were positioned in the same module and Latvia, in the end, was itself a cluster.

These results allow the researcher to conclude that territorial disposition still has a great impact in the choice of countries to select a trade partner. Both import and export graphs reflect similar clustering groups – which is reasonably acceptable once we are analysing the same group of countries. Moreover, if country A sells to country B, correspondingly country B bought from country A. Here also HDI fairly reflects in the same extent the level of export trades. We can also say that, considering the order of clusters, economic cooperation regarding inter-dependency is higher for the most recent members in the EU (mainly East Europe countries), when compared, for instance, with the founding members of EU (mostly Central and Southern Europe). Denmark, Finland and Sweden seem not to have relevant trade interaction with the rest of Europe as they seem to have good trade relations among them. These Northern countries might be good examples of self-sustainability in terms of commercial exchanges. Lastly, clusters composed by Estonia and Lithuania and Latvia might indicate that they are still not well-integrated in the context of the European single market.

Centrality Measures

Considering the centrality measures for the export variable, similarly to the import trade analysis, the researcher wanted to attest which countries export the most, which in methodologically terms it means we are interested in analysing out-degree outcome. By using *R*, the **Table 26** shows the out-degree measure sorted in ascending order in terms of exports in 2011, compared with HDI Rank. From the analysis of **Table 26**, the researcher could find a trend for most developed EU countries (with higher HDI) having more export capability than emerging countries. More developed economies are better connected than developing countries (Germany, Netherlands, Belgium and Sweden are examples of densely connected countries). Germany is, among all EU-28 countries, the one with highest export volume with more than 50% than the second highest exporter (France).

Considering the outcomes of betweenness and closeness metrics (**Table 26**), when considering exclusively connections between countries, Finland and Poland are the countries with highest number of links in the network; on the other hand, Luxembourg, Malta, Estonia and Latvia are the ones with less interaction with the rest of Europe. If only weights (export volume) are considered, betweenness confirms the supremacy of Germany in terms of export trades. Closeness centrality does not show any important conclusion when considering only ties between countries as all outcomes were identical for all countries. When considering only weights, results are again similar, with the highest output coming from Germany, meaning that Germany is the most attractive country in EU-28, in terms of exports.

b.2.1. Balance of Trade (BOT)

An important statistical tool to understand the relative strength of a country's economy versus other countries' economies and the flow of trade between nations is the Balance of Trade (BOT). In that sense, the researcher compared outcomes from out degree for both Import and Export data in 2011. The following graph shows then the difference between each EU-28 country's imports and its exports for the year 2011. Germany leads the rank. In 2011, it had the largest trade surplus in EU-28, followed by Netherlands. Countries like Ireland, Slovakia and Czech Republic had as well a positive trade balance. On the opposite side, United Kingdom was in 2011 the country with highest trade deficit of around 100 million euros. This inference can be an important asset considering the current Brexit situation. France, Sweden and Greece were also other examples of negative trade balance for the same period.

Despite the results, a trade surplus or deficit, taken on its own, is not necessarily a viable indicator of an economy's health.

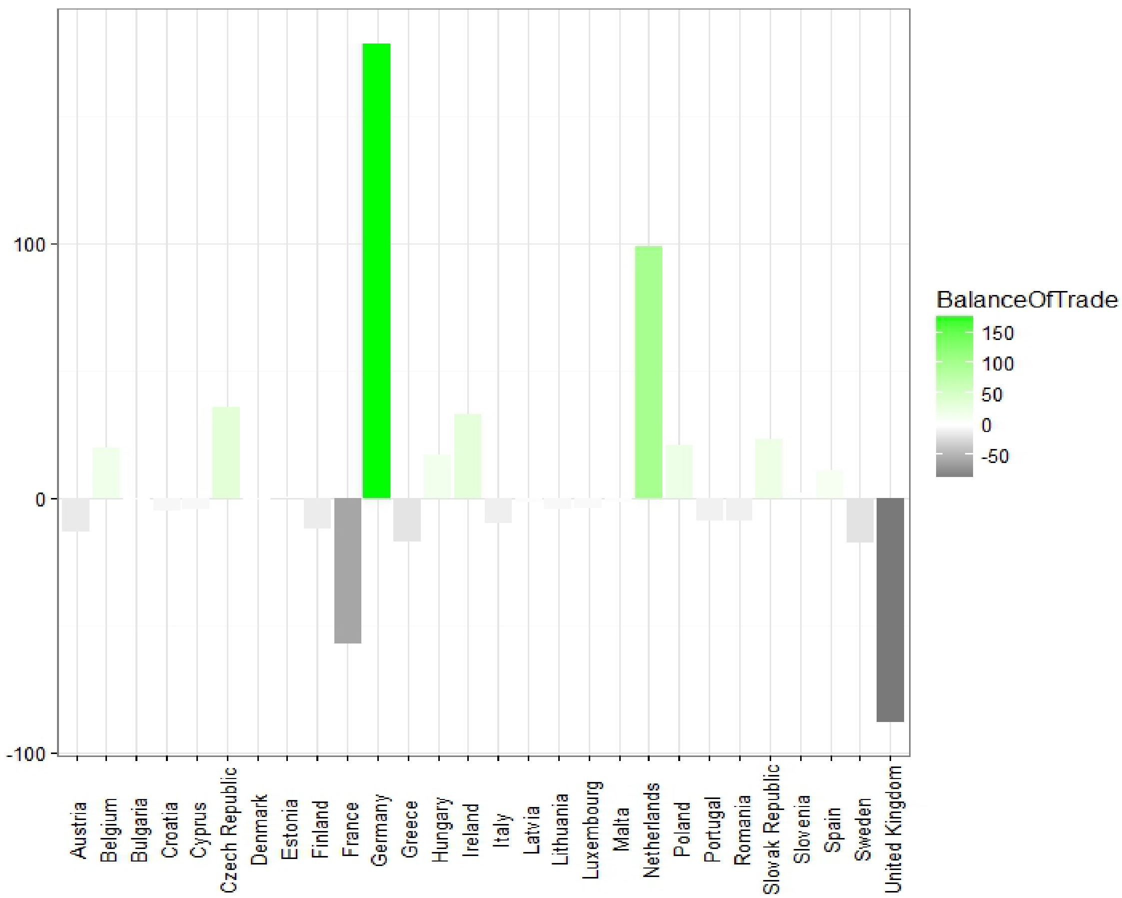


FIGURE 20 – EU Domains - Balance of Trade (BOT) (in millions €) **SOURCE:** Author, R

b.3. FDI FINANCIAL FLOWS

Foreign direct investment (FDI) is defined as an investment involving a long-term relationship and reflecting a lasting interest and control by a resident entity in one economy (foreign direct investor or parent enterprise) in an enterprise resident in an economy other than that of the foreign direct investor (UNCTAD, 2009).

"FDI data is an essential tool for research and policy analysis, and a basis for policy formulation, implementation and assessment. In fact, the scarcity, unreliability and inconsistency of FDI data pose a serious challenge for policy-makers, academics and practitioners", said James Zhan, Director of Investment and Enterprise at UNCTAD (2014)[12].

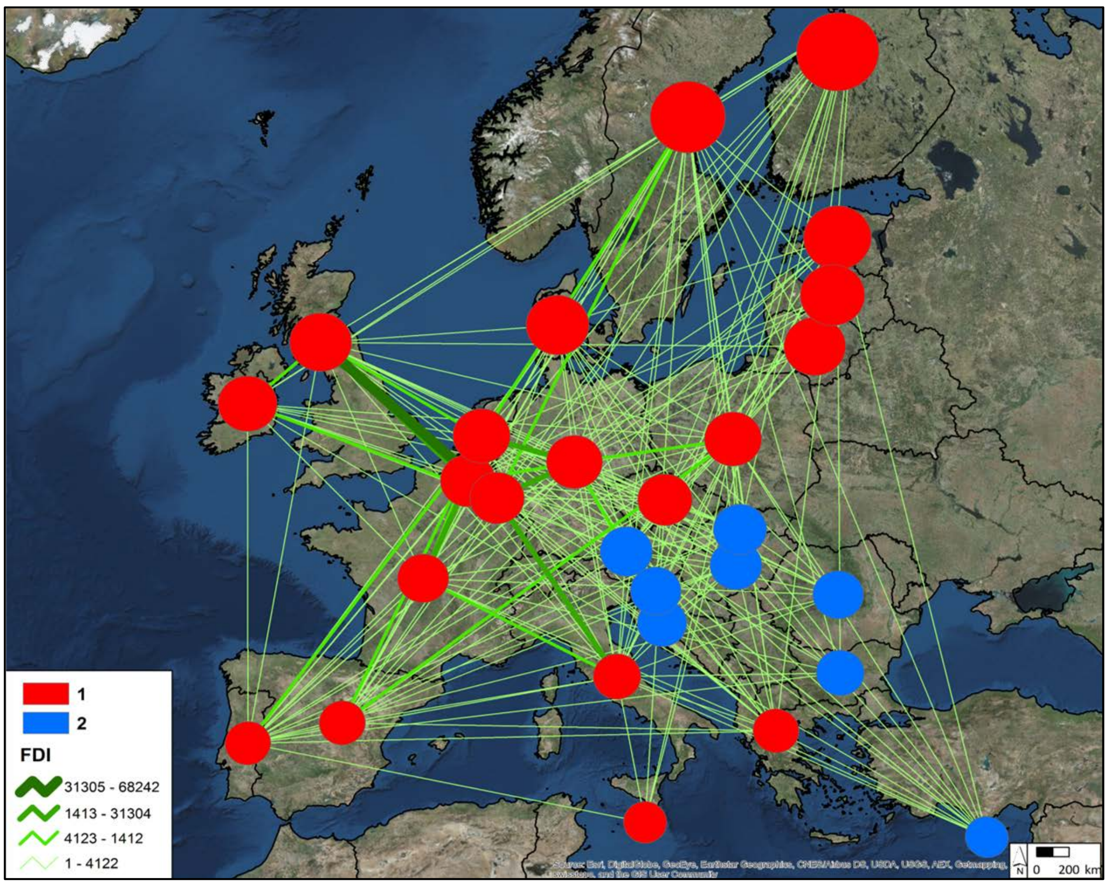


FIGURE 21 – EU Domains – FDI: Community network **SOURCE:** Author, ArcGIS

[12]Available at: unctad.org/en/Pages/DIAE/FDI%20Statistics/FDI-Statistics-Bilateral.aspx

The following graph expresses the obtained territorial network for the EU-28 considering foreign direct investment data for the year 2011. Each link represents an investment made from one country to another; each vertex denotes a country. Edge colours denote densely connected subgraphs (community detection method).

In terms of FDI, EU countries were split into 2 different clusters:

[1] Belgium, Denmark, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Spain, Sweden and United Kingdom

[2] Austria, Bulgaria, Croatia, Czech Republic, Hungary, Slovakia, Slovenia and Romania

This network clustering does not establish a specific partitioning of EU countries. Seven countries were kept apart from the remaining Europe. These countries include a EU founder (Netherlands), some emerging countries of East Europe, the most recent EU member (Croatia) and a central Europe country (Poland). In economic terms, these results can have different reasons to any country. Then we proceed by analysing centrality measures.

CENTRALITY MEASURES

For the FDI analysis, two different approaches can be considered. We can either analyse outcomes in the perspective of trying to realize who the countries with highest international flow investments are either to figuring out which are the most attractive countries for investment purposes. In methodological terms, we can look for both **FIGURE 20** and **TABLE 27** to derive these two perspectives. Italy constitutes the country with higher volume of received investment from other EU countries in 2011, followed by Spain and Lithuania. In terms of connections, besides Italy and Spain, also Latvia, Belgium and Finland belongs to the top five countries with higher number of countries adjacent to it. Regarding investment performed in other EU members, in 2011 Spain and Belgium were the countries that spend the most in terms of foreign direct investment, with total investment amounts higher than 90.000 thousand USD. No causality relationship seems have between EU ranked countries by In-Degree and FDI variable with HDI rank countries in 2011. Looking into detail to the betweenness and closeness outcomes:

- On the one hand, for the betweenness metric, when considering only ties between countries, Latvia and Finland confirm to be the better-connected countries in the FDI network; when considering the investment volume instead of only the ties themselves, Spain, Finland and Belgium are the most important countries in terms of connecting the all network
- On the other hand, closeness centrality shows that Italy is the country presenting the highest amount of foreign direct investment. Indeed, **FIGURE 22** confirms that Spain was the country among all that invest the most to Italy. Denmark has the highest closeness value when considering only the investment volume: mostly because it constitutes a country that only receives investment (once in that case, no data was available to this country).

With regards of reciprocity measure, *R* computations showed that there is about 43,6% percent chance that if a given country A invests in another country B, then that other country B also invests back to country A.

b.4. BILATERAL REMITTANCES

The movement of people across international boundaries has enormous implications for growth and poverty mitigation in both origin and destination countries. In result, "remittances are a vital source of financial support that directly increases the income of migrants' families. Remittances lead to more investments in health, education, and small business" said Hans Timmer, current Chief Economist at the World Bank (2010) [13].

[13] Available at: go.worldbank.org/AOQONKFW80

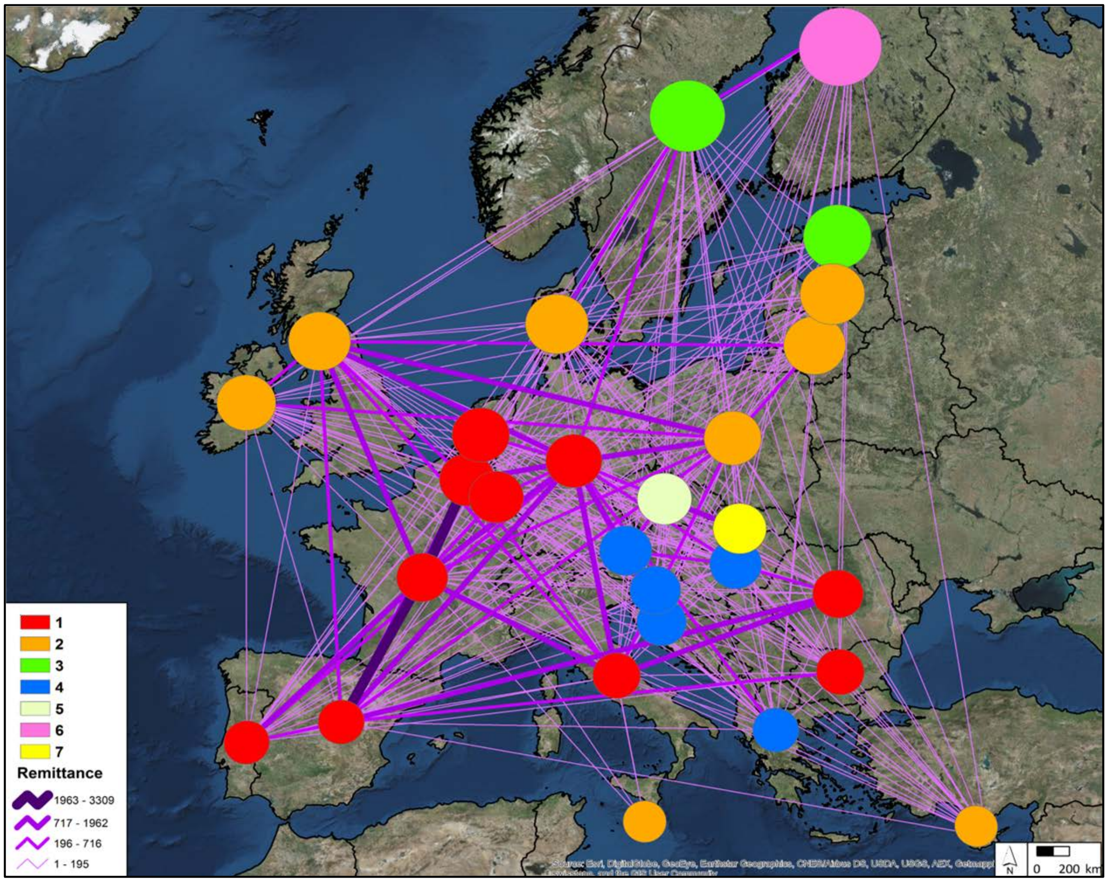


FIGURE 22 – EU Domains - Remittances: Community network **SOURCE:** Author, ArcGIS

The notion of remittances includes the sum of two main components:

- Compensation of employees – "remuneration in return for the labour input to the production process contributed by an individual in an employer-employee relationship with the enterprise." and
- Personal transfers – all current transfers in cash or in kind made or received by resident households to or from non-resident households.

Figure **19** represents the network obtained for EU-28 bilateral remittances data in 2011. Each vertex represents a EU country; each edge characterizes remittances sent from one country to another. Edges are grouped by colour, under the community detection concept: countries with the same colour are densely connected subgraphs (short random walks tend to stay in the same community).

Regarding Bilateral Remittances data, EU countries were clustered in 7 different groups:

[1] Belgium, Bulgaria, France, Germany, Italy, Luxembourg, Netherlands, Portugal, Romania and Spain;

[2] Cyprus, Denmark, Ireland, Latvia, Lithuania, Malta, Poland and United Kingdom;

[3] Estonia and Sweden;

[4] Austria, Greece, Croatia, Hungary and Slovenia;

[5] Czech Republic;

[6] Finland;

[7] Slovakia.

High-income EU countries are the main source of remittances. Countries like Germany, France, Luxembourg and Netherland positioned in the first cluster. However, other emergent countries such as Bulgaria and Romania were also good remittances sources in 2011. Denmark, Ireland and United Kingdom are examples of countries ranked in the second largest cluster, followed by Estonia and Sweden and Finland. The fifth cluster included some central and south European countries and Check Republic and then Slovakia were the economies less prone to send remittances to another EU country in 2011. A closer observation to **FIGURE 22** indicates that globally, Germany and France were the largest sources of remittances in EU in 2011, both sending and receiving. In particular, there is a strong remittance connection between Belgium → France, France → Spain, and Spain → France.

Observing more into detail outcomes from **TABLE 28**:

- HDI and in and out degree measures do not seem to have a strong linear correlation;
- Betweenness outcome ranks Greece, Czech Republic and Germany as the most central countries emerging in a path in terms of number of ties; on the other hand, Croatia, Poland and Bulgaria are countries with limited large influence on the transfer of items through the network. In terms of volume of remittances sent, Czech Republic is also the best positioned country, followed by Estonia and Germany.

- Closeness centrality outcome put Germany and United Kingdom, Spain and France as the most central nodes of the network, contrarily to Lithuania, Estonia or Bulgaria., both in terms of ties and weights.

Reciprocity measure for bilateral remittance variable was, by *R* calculation, 76,5%. This outcome implies that there is about 76,5% probability that if a given country A is linked for remittances purposes with another country B, then country B also links back to country A.

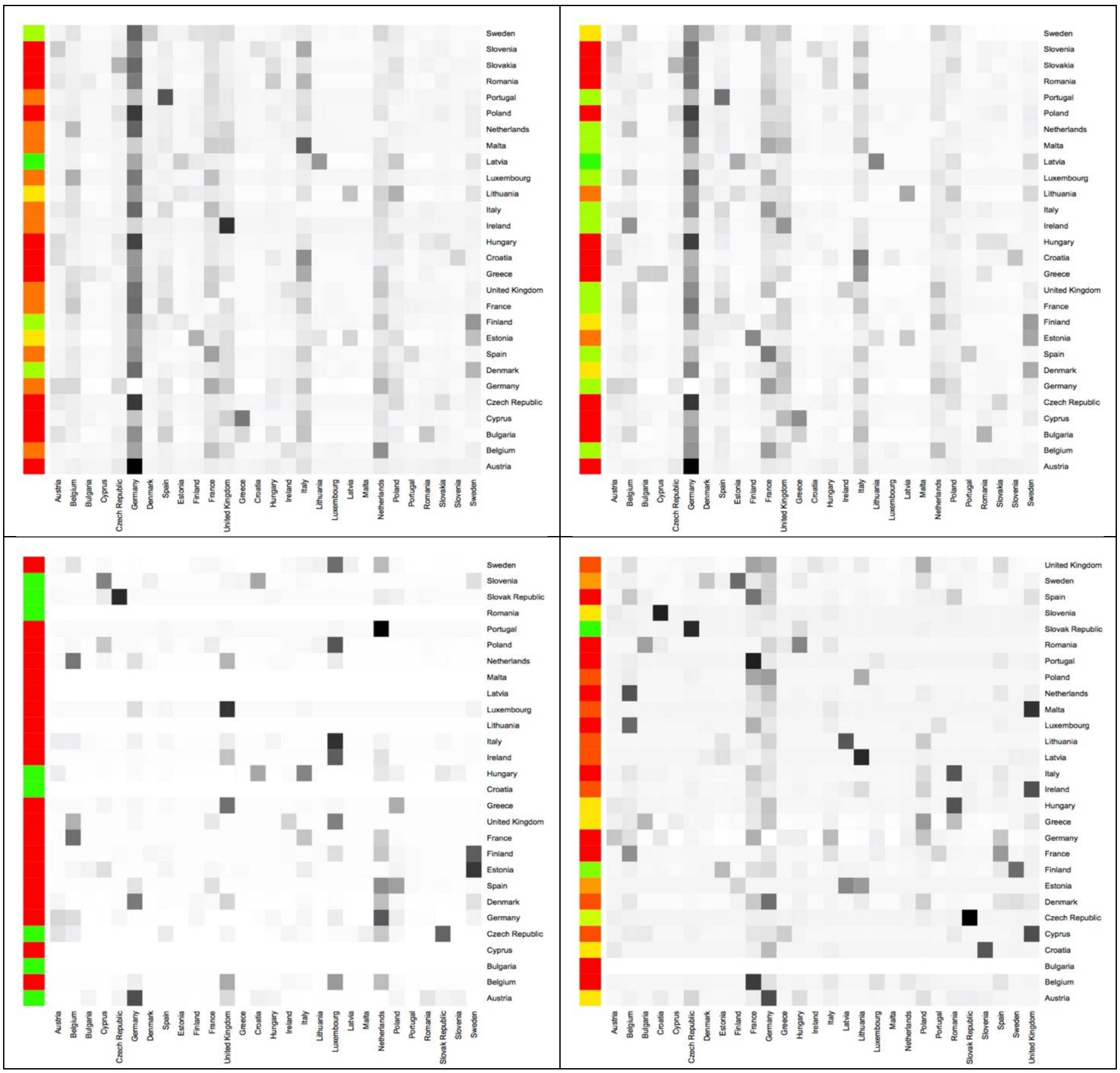


FIGURE 23 – EU Domains - Heat maps (Imports, Exports, FDI and Bilateral remittances) **SOURCE:** Author, R

3.3. MULTIPLEX NETWORK ANALYSIS

In the multiplex network analysis, the researcher aggregates the different monolayers to represent the complex structures of each one of the case studies and derive the main conclusions. Both cases are examples of interconnected multiplex networks, with the following characteristics:

- **(i)** The intra-layer edges are coloured;
- **(ii)** Any pair of layers has at least one node in common;
- **(iii)** The inter-layer links connect the replicas of each node across layers.

a. PORTUGUESE URBAN SYSTEM ANALYSIS

a.1. DIAGNOSTICS

Regarding the Portuguese urban system application, this thesis examines the two mono-layers in an aggregate level. The table below shows the basic statistics characterising each one of the mono-layer network composing the overall multiplex network.

		WORK LAYER	ACADEMIC LAYER
# Nodes		308	308
# Edges		21988	9006
Density		71,4	29,2
# Components	Weak	1	1
	Strong	3	3
Diameter		5	13
Mean Path Length		1.8	2.2

TABLE 10 – PT Urban System - Multiplex Analysis: Diagnostic measures **SOURCE:** Author, Muxviz

All nodes are represented in both mono-layer networks. Despite of that, commuting due to work reasons happens more frequently than due to academic reasons: among all the considered links, around 70% of them were related to working purposes. As for the network density, describing the portion of the potential connections (connection that could potentially exist between two “nodes”

– regardless of whether or not it actually does) that are actual connections, the highest percentage is for the work layer when compared to the academic layer. This can possibly be related to the fact that there are more working locations rather than school facilities. Observing the entire components structure, both for the work and academic layer, the networks are fully connected, although, there are three densely connected sub-components, being Porto, Lisbon and Algarve regions. Furthermore, one can induce that the average geodesic distance among the municipalities is quite small, suggesting a network in which commuting is likely to occur in every municipality, and to do so quickly. Indeed, on the academic layer, it is necessary 13 steps to get from one side of the network to the other, while for the working layer, the largest geodesic distance (diameter) is 5 steps. The mean path length gives similar conclusions: the average number of steps along the shortest paths for all possible pairs of the work layer nodes is 1.8 and of the academic layer is 2.2.

The basis diagnostics of the Portuguese urban system gives an overall idea that the working layer has higher share, both in volume and in density, for the multiplex network analysis.

a.1.1. CORRELATION

The following table further explore the two mono-layer networks, by showing the outcome of some measures of similarity between layers.

	CORRELATION WORK / ACADEMIC LAYERS
Mean global overlapping (Node / Edge)	63.52 %
Inter-layer Assortativity: Pearson	85.30 %
Inter-layer Assortativity: Spearman	89.50 %

TABLE 11 – PT Urban System - Multiplex Analysis: Correlation **SOURCE:** Author, Muxviz

63.52% is the fraction of edges which are common to both layers. Moreover, the Pearson/Spearman inter-layer assortativity outcomes show that the correlation between the degree (strength) of nodes and their counterparts in other layers, for all pairs of layers is, respectively 85.30% and 89.50%. As such, the obtained outcomes by the inter-layer assortativity indicate strong linear correlations between the work and academic layers is strong and positive.

a.1.2. COMMUNITY

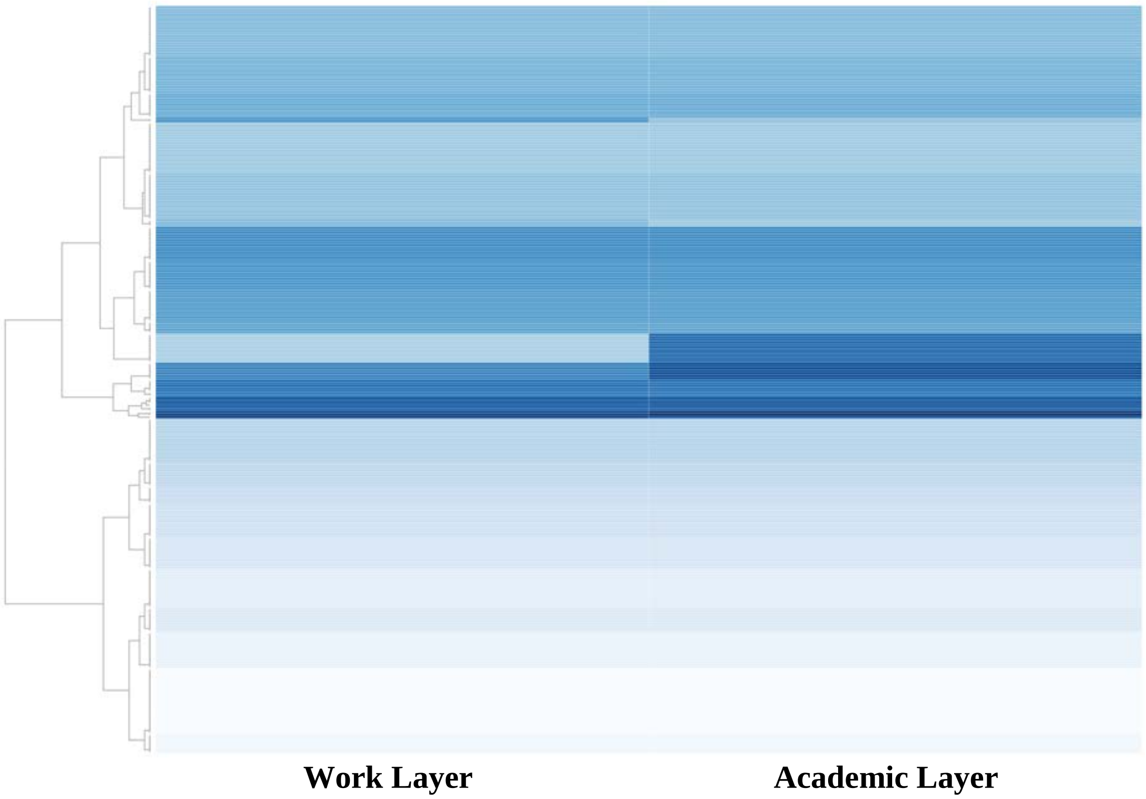


FIGURE 24 – PT Urban System - Multiplex Analysis: Community **SOURCE:** Author, Muxviz

In general, the figure above suggests that the community detected by the random walktrap algorithm is slightly structured in a similar way in both work and academic layers. The computation of the aggregate community structure is presented as follows:

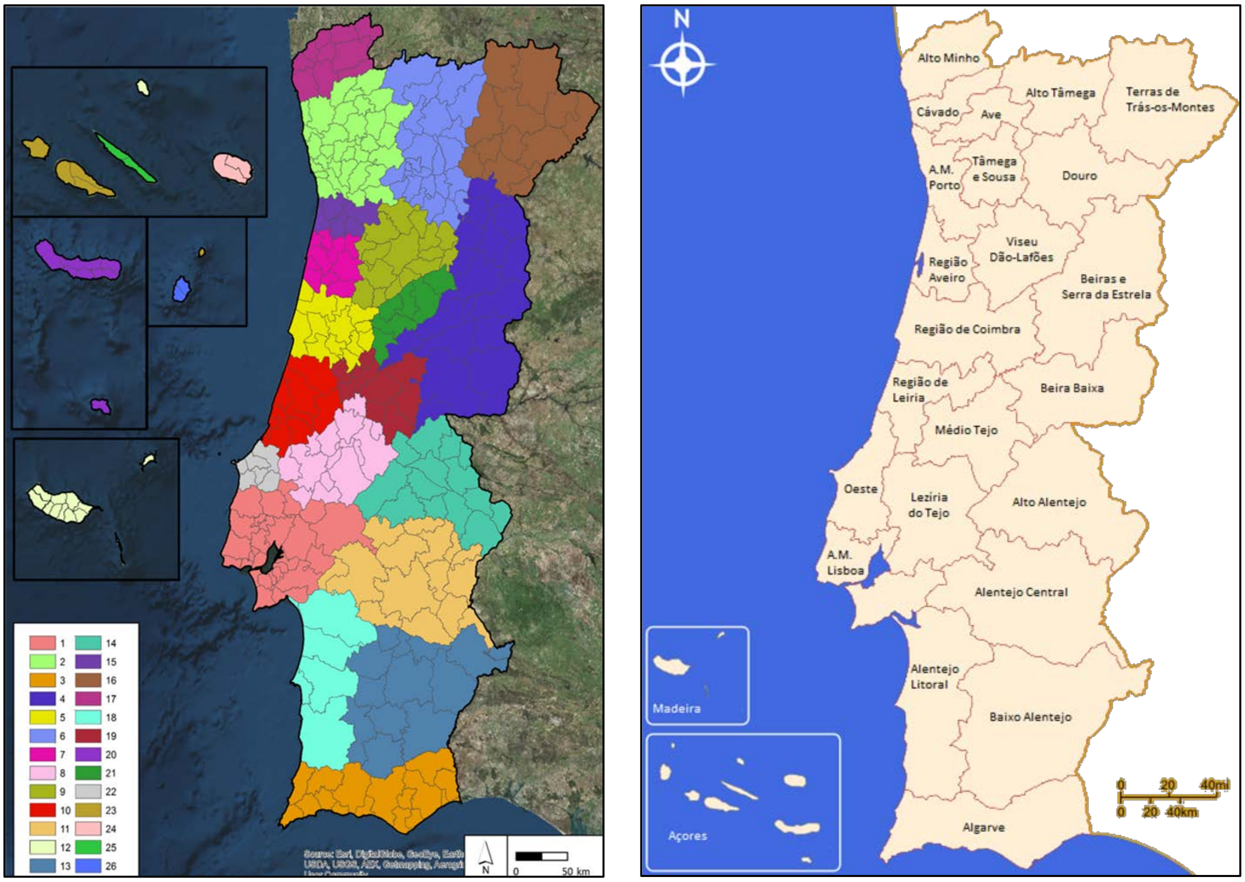


FIGURE 25 – PT Urban System - Multiplex Analysis: Aggregate Community vs NUTS III **SOURCE:** Author

Despite of the diversity/heterogeneity of the approaches and perspectives, it can be demonstrated that there is a common denominator in the multilayer clustering, represented in the figure above: the geographic proximity. Indeed, the geographic proximity seems to be the crucial key driver to network formation both for labour-force and student population, regardless of the Portuguese municipalities size, composition or economic sphere. To a certain extent, the multilayer clustering outlines similarities with the Portuguese urban system division in NUST III, especially in the south of Portugal. NUTS III organisation divides Portugal in 30 territorial units, 28 on the Mainland, the Autonomous Region of the Azores and the Autonomous Region of Madeira. As such, the methodology chosen might be a good analysis instrument for defining or restructuring the national municipality's master development plan.

b. European Union Domains Analysis

b.1. Diagnostics

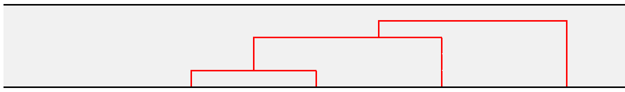
	Imports Layer	Exports Layer	FDI Layer	Remittances Layer
# Nodes	28	28	28	28
# Edges	560	560	280	594
Density	20	20	10	17.6
# Components	1	1	1	1
Mean Path Length	1.3	1.3	1.5	1.3

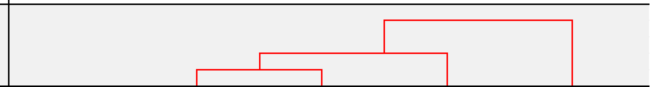
Table 12 – EU domains - Multiplex Analysis: Diagnostic measures **Source:** Author, Muxviz

The table above denotes the basic diagnostics of each monolayer and respective multiplex network, if applicable. The measured statistics are the number of nodes, edges and components and the mean path length. The outcome allows the researcher to conclude that each layer is composed by the 28 European Union countries and is fully connected in one single component.

Moreover, each pair of nodes is, on average, connected in a one-to-one basis as the average number of steps along the shortest paths for all possible pairs of network nodes is 1.3 for Imports, Exports and Remittances monolayers and 1.5 for the FDI monolayer. The average number of hops that it takes to reach every other node is then short.

b.1.1. CORRELATION





	INTER-LAYER ASSORTATIVITY: PEARSON					INTER-LAYER ASSORTATIVITY: SPEARMAN			
	IMPORTS	EXPORTS	REMITTANCES	FDI		IMPORTS	EXPORTS	REMITTANCES	FDI
IMPORTS	1	0.945	0.842	0.753	IMPORTS	1	0.965	0.873	0.813
EXPORTS	0.945	1	0.834	0.728	EXPORTS	0.965	1	0.881	0.761
REMITTANCES	0.842	0.834	1	0.82	REMITTANCES	0.873	0.881	1	0.773
FDI	0.753	0.728	0.82	1	FDI	0.813	0.761	0.773	1

TABLE 13 – EU domains - Multiplex Analysis: Correlation (Colour representation) **SOURCE:** Author, Muxviz

The outcomes proposed by the Spearman's inter-layer assortativity indicate strong linear correlations between each pair of the considered mono-layers, with coefficients higher than 75%. In particular, the association between Imports and Exports layers is close to be considered a perfect monotone function, as the correlation coefficient is 0.965.

Furthermore, the outcomes processed by the Pearson's inter-layer assortativity assess positive linear correlations between each pair of layers.

b.1.2. COMMUNITY

The following graph unveils the community structure of the EU domains network. By applying the multiplex map algorithm, proposed by Pons and Latapy (2005), the community detection is found by trying to identify densely connected subgraphs via random walks.

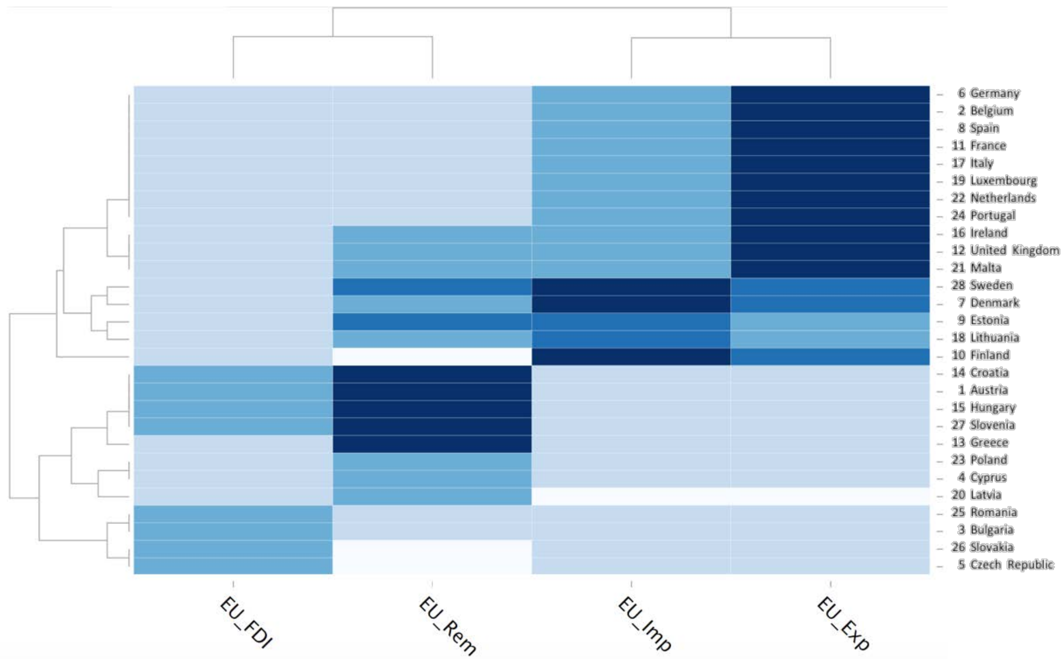


FIGURE 26 – EU domains - Multiplex Analysis: Community (Colour representation) **SOURCE:** Author, Muxviz

Similarly to previous conclusions, the compilation of the individual communities of each EU domains layer shows that's the layers related to Imports and Exports have the same composition. Besides that, when applying the clustering, Exports and Imports layers are gathered together, just as FDI and remittances layers.

The aggregate community detection suggests clustering the EU-28 countries as follows:

[1] Latvia;

[2] Austria, Bulgaria, Cyprus, Czech Republic, Greece, Croatia, Hungary, Poland, Romania, Slovakia and Slovenia;

[3] Estonia and Lithuania;

[4] Belgium, Germany, Spain, France, United Kingdom, Ireland, Italy, Luxembourg, Malta, Netherlands and Portugal;

[5] Denmark, Finland and Sweden.

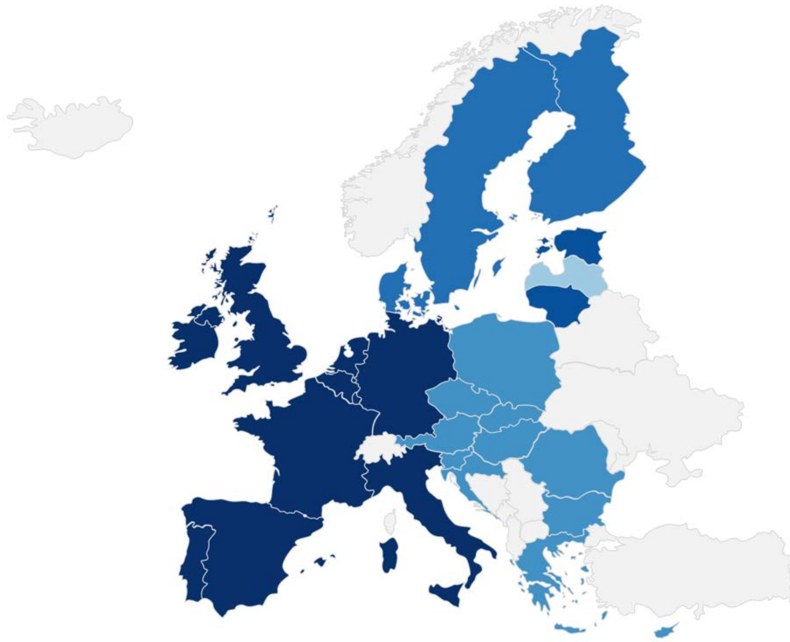


FIGURE 27 – EU domains - Multiplex Analysis: Aggregate Community (Colour representation) **SOURCE:** Author

In general, the EU founding members are positioned on the same cluster, together with members that participate on primary enlargements of the European Union expansion. Indeed, the main cluster includes the 6 EU founding fathers (Belgium, Germany, France, Italy, Luxembourg and the Netherlands) and countries that participated in the following European Union enlargements, namely Ireland and the United Kingdom (1973), Greece (1981) and Spain and Portugal (1986). As a result, the longer a country member have belonged to the European Union, the more integrated / connected in the network it stands, particularly when assessing the trade, investment and remittances between the EU-28 countries.

Another relevant cluster groups Nordic European countries i.e. Sweden, Denmark and Finland. This cluster contributes to the universal fact that countries have closer relationships with their neighbours.

Most of the countries integrating the EU in 2004 and eastern countries like Romania and Bulgaria (2007) and Croatia (2013) mainly constitutes the third aggregate cluster. The EU membership can be considered as an ongoing prospect into the world economy and a powerful transformative effect. This is particularly applicable to the aspiring countries incorporated in this cluster.

Two additional clusters are identified: Estonia and Lithuania and Latvia itself. These are examples of developed countries with less capability of facing the EU competitive environment. Integrating such large economy triggers democratic, political, economic and societal changes. The Baltic nations are still in an early stage of the integration in the European single market, with fewer facility to systematically react and be in tune with the remaining EU.

Besides structuring the countries into 4 main different clusters, the model determines a modularity of **0.052**. The modularity of a graph is a goodness factor of partition, measuring how separated are the different vertex types from each other. Such small value explains that the aggregate network is densely linked and no specific country is disconnected. The composition of the multiplex network nearly reflects then one of the EU's founding principles, to encourage the free movements of people, goods, services, and money among its members.

b.2. VISUALISATION

Toward the European Union domains, the researcher aggregates the four-considered mono-layers and obtain the following graph representation:

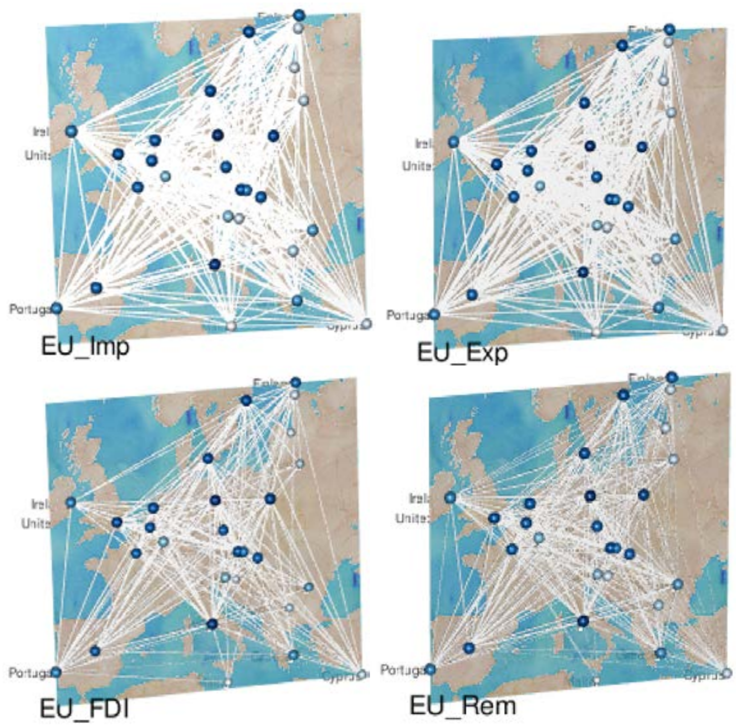


FIGURE 28 – EU domains - Multiplex Network: Visualisation **SOURCE:** Author, Muxviz

b.3. Reducibility

Structural Reducibility

	Exports	Imports	Remittances	FDI
Exports	0	0.078	0.225	0.354
Imports	0.078	0	0.204	0.347
Remittances	0.225	0.204	0	0.35
FDI	0.354	0.347	0.35	0

Reducibility Dendrogram

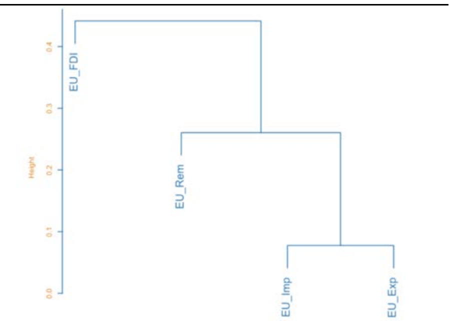


Table 14 – EU domains - Multiplex Analysis: Reducibility (Colour representation) **Source:** Author, Muxviz

It can be verified that the multiplex network could be reduced to 3 layers without relevant costs by "merging" the Imports and Exports layers, as the impact is approximately zero (0.078). The remittances layers would follow to reduce the dimension of the multiplex to 2 layers and followed by the FDI layer. In general, the mono layers appear to be relatively similar, since the maximum identified value for the reducibility is 0.35 (between FDI and Remittances layers).

b.4. Nodes Centrality

Based on the degree centrality descriptor (sum of in-degree and out-degree flows), the centrality of the European Union countries is identified as follows:

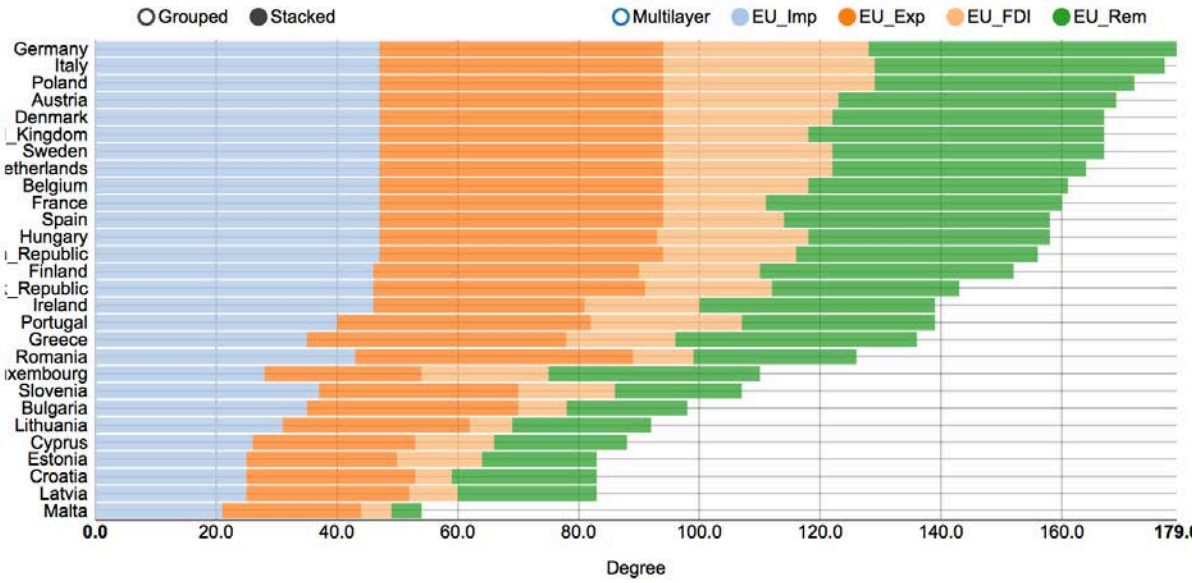


FIGURE 29 – EU Domains - Node Centrality by degree descriptor **SOURCE:** Author, Muxviz

The graph above shows the European Union countries ranked by the sum of the in and out degree outcomes in the different monolayers. Overall, the most significant European countries with highest number of imports, exports, FDI and remittances flows are Germany, Italy, Poland, Austria and Denmark. On the bottom of the list, countries like Estonia, Croatia, Latvia and Malta are the less represented one. In conclusion, the multilayer results (sum of the monolayer outcomes) most likely reflect the obtained outcomes of each individual monolayer.

b.5. RANKING RESULTS

In this subsection, the researcher applies another method to identify the most central EU-countries of the multilayer network and conduct a comparison of the obtained individual ranking results from the four main considered centrality measures (degree, betweenness, closeness and eigenvector) with the ranking result according to gross domestic product (GDP) and Human Development Index (HDI) (see **TABLE 15**). The following assumptions are taken into consideration:

- The rankings of the node degree, betweenness and closeness are the result of computing a weighted average of the respective individual outcomes. Considering the analysis performed in the reducibility section, where the Imports and Exports layers turned out to be strongly redundant and correlated, the author decides to partition 1/6 for the individual results of Imports and Exports layers and 1/3 to the individual outcomes of FDI and remittances. The betweenness and closeness individual scores are based on tie weights only (alpha = 1);

- The eigenvector measure is computed to the aggregate network (in *Muxviz*).

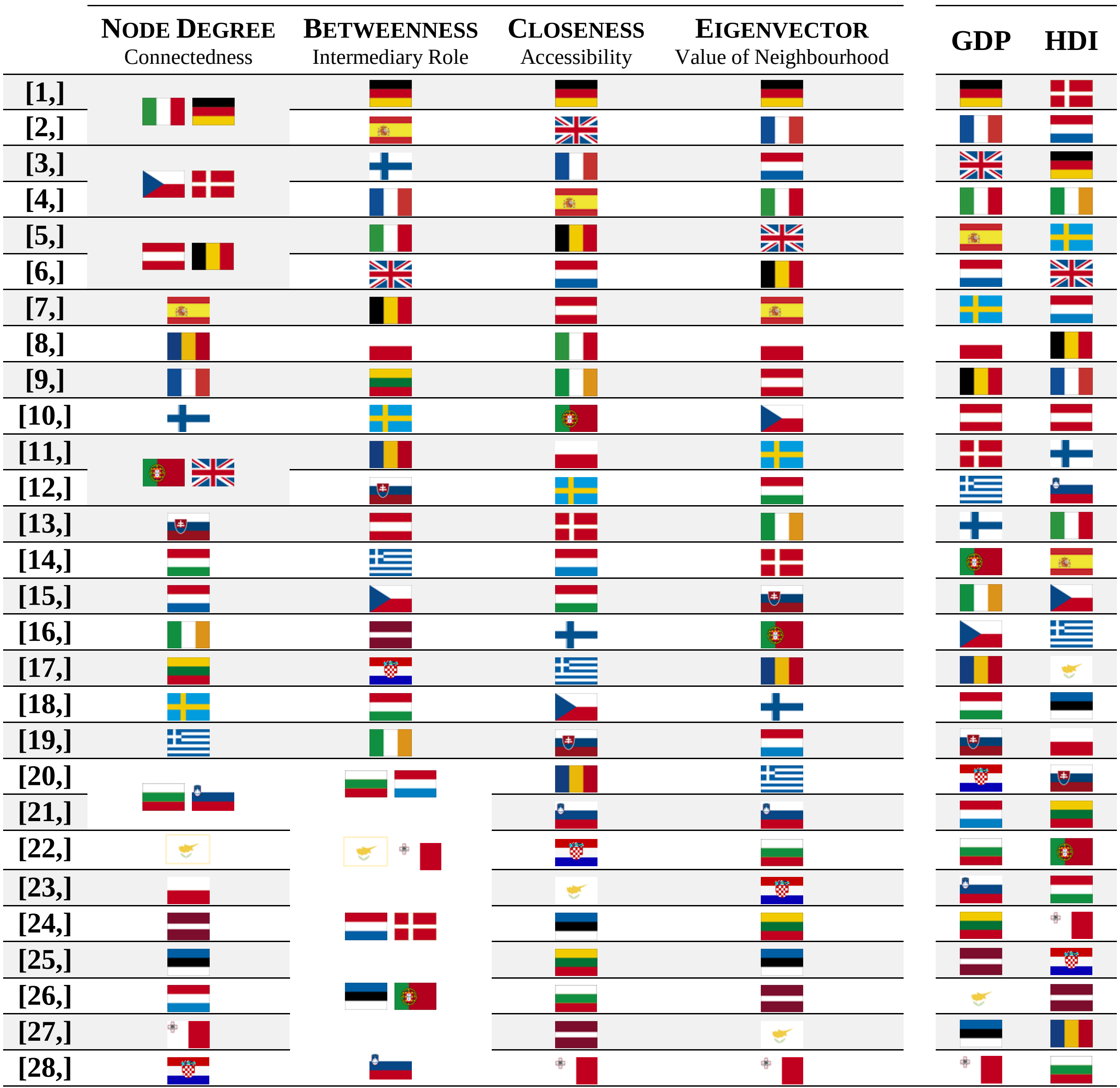
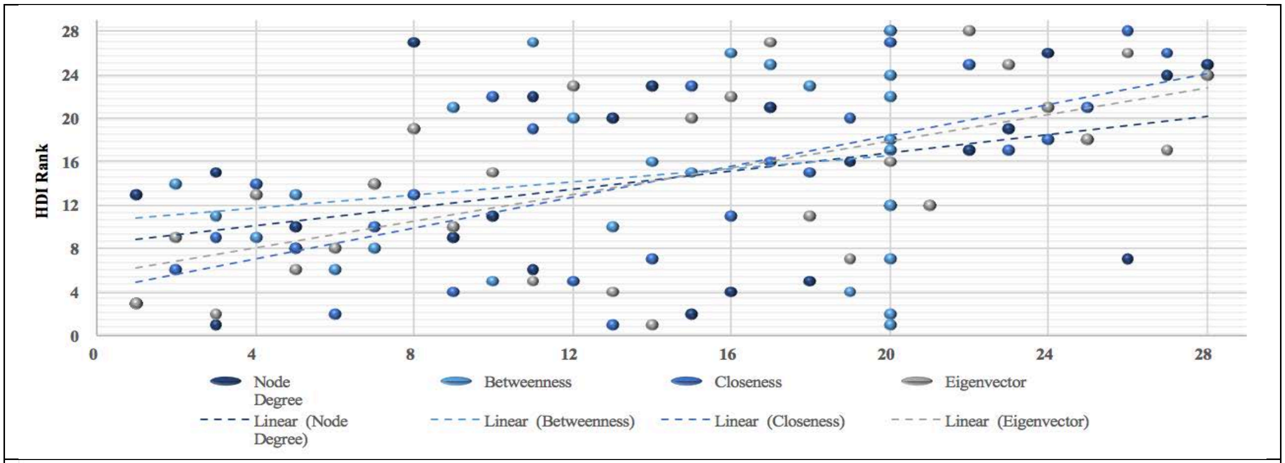


TABLE 15 – EU Domains – Ranking by centrality measures, GDP and HDI – **SOURCE:** Author

The ranking results show the remarkable role of Germany in leading the European Union for the different centrality measures, followed by mainly by other big economies such us France, Italy, Spain and Belgium. On top ten, the country like United Kingdom, the Netherlands, Austria and Finland are also on top ten of the most central countries.

The charts below compare the individual ranks of each centrality measure with the ranking for the HDI, in the first graphic and GDP in the second one.



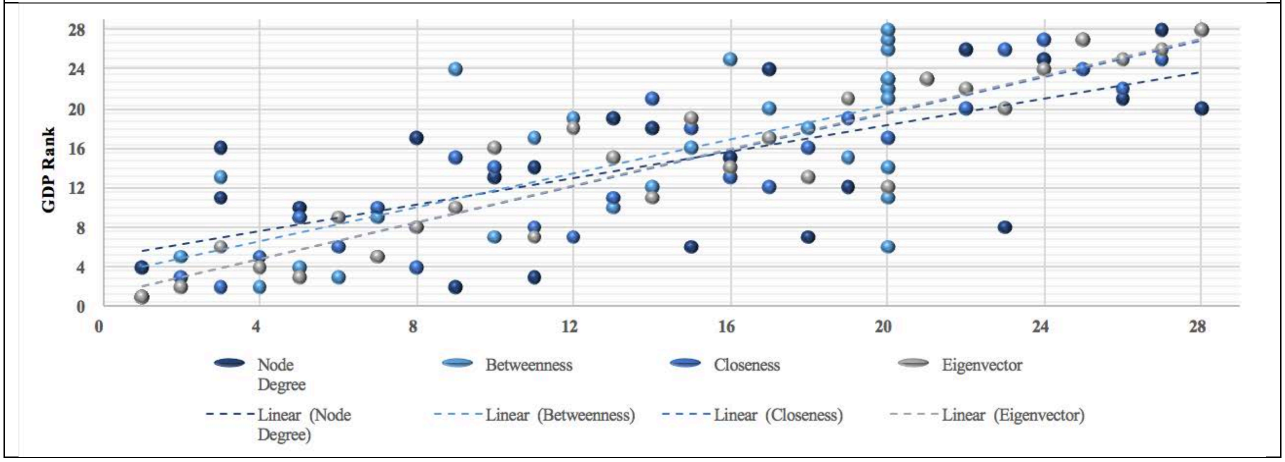
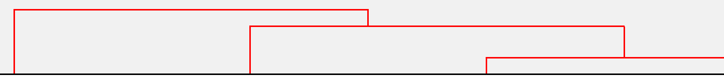


FIGURE 30 – EU Domains - Centrality Measures vs HDI / GDP **SOURCE:** Excel, Author

The four centrality measures have in general ranked the EU-28 countries similarly to the ranking result according to HDI, exhibiting a positive correlation. The same applies to the comparison with the GDP variable.

In order to examine the level of similarity between each pair of centrality measures with respect to all ranking results, the researcher calculates Kendall's tau-b coefficients for each pair of the measures (see **TABLE 16**).

KENDALL'S TAU COEFFICIENT



	BETWEENNESS	**DEGREE**	**CLOSENESS**	**EIGENVECTOR**
BETWEENNESS	1	0.403	0.395	0.478
DEGREE	0.403	1	0.455	0.519
CLOSENESS	0.395	0.455	1	0.773
EIGENVECTOR	0.478	0.519	0.773	1

TABLE 16 – EU domains - Multiplex Analysis: Kendall's coefficients (Colour representation) **SOURCE:** Author, R

Kendall's tau coefficient is a measure of ordinal association between two variables, and takes values between −1 and 1. A coefficient of 1 denotes that the two variables rank data in exactly the same order, and a coefficient of −1 implies that the two variables rank data in exactly the reverse order.

From **TABLE 16**, the pair of centrality measures that give very similar but not exactly the same ranking orders is Eigenvector–Closeness. All pairwise Kendall's tau-b coefficients exhibit positive similarity. This suggests that all centrality measures are relatively aligned in ranking similarly the EU-28 countries, which matches with the conclusions identified in **FIGURE 28**.

3.4. MINING NETWORKS

In this section of Chapter 3, the author uses an inductive method and formulate supervised learning problems for each one of the previous depicted case studies.

The main goal is to classify the Portuguese municipalities / EU-28 countries by identifying patterns (between municipalities and between countries), from where it is possible to predict their betweenness centrality in the network, according to other important input attributes. The chosen technical solution consists of implementing Naive Bayes and Decision Trees classifiers.

- Naive Bayesian classification is based on Bayes' theorem of posterior probability and assumes that the effect of an attribute value on a given class is independent of the values of the other attributes. This assumption is called class-conditional independence.
- Decision tree induction is a top-down recursive tree induction algorithm, which uses an attribute selection measure to select the attribute tested for each non-leaf node in the tree. Each branch represents an outcome of the test, and each leaf node (or terminal node) holds a class label. The topmost node in a tree is the root node.

The target attribute (output) is built in a two-step approach: firstly, the weighted average of the mono-layers betweenness outcomes is computed per municipality/country (same as in section 3.3); secondly, the obtained numeric data is discretised, by mapping the values to interval or concept labels. The betweenness centrality is chosen as target attribute because, among all the considered centrality measures in section 3.4, it was the one exhibiting a more positive correlation according to the GDP and HDI variables.

a. PORTUGUESE URBAN SYSTEM ANALYSIS

Given the Portuguese municipalities, data is collected from *Statistics Portugal* (reference year: 2011). The considered input attributes are as follows:

- Council housing dwellings (No.); Resident population in census localities (No.); Proportion of resident population with higher education completed (%) by sex; Unemployment rate (%) by sex.

The target attribute (betweenness centrality) discretization process is based on the following groups:

- "High" for the top 53 municipalities with highest averages of the 2-layer betweenness outcomes: 18% of all municipalities represent 82% of overall betweenness volume, which approaches the 20/80 assumption of the Pareto analysis very closely;
- "Low" for municipalities with average scores of the 2-layer betweenness outcomes equal to zero;
- "Medium", otherwise.

a.1. DECISION TREE CLASSIFIER

The default percentages of 30%/70% are used to partition data into test and train data, respectively. The following decision tree and further results are shown below:

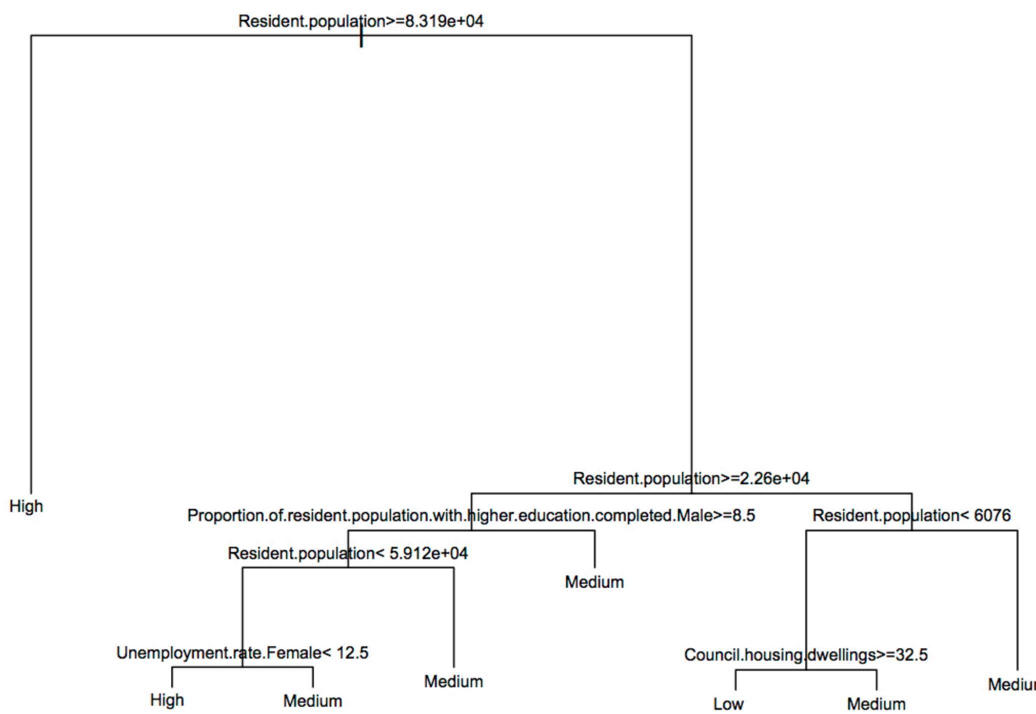


FIGURE 31 – PT Urban System - Decision Tree: Representation **SOURCE:** Author, R

		Predictions		
		High	**Medium**	**Low**
Correct	**High**	4	8	0
	Medium	2	52	6
	Low	0	16	5

Accuracy	0.6559
95% CI	(0.5502, 0.7514)
No Information Rate	0.8172
P-Value [Acc > NIR]	0.9999

Kappa	0.2139
Mcnemar's Test P-Value	NA

Statistics by Class	Class: High	Class: Low	Class: Medium
Sensitivity	0.66667	0.45455	0.6842
Specificity	0.90805	0.80488	0.5294
Pos Pred Value	0.33333	0.23810	0.8667
Neg Pred Value	0.97531	0.91667	0.2727
Prevalence	0.06452	0.11828	0.8172
Detection Rate	0.04301	0.05376	0.5591
Detection Prevalence	0.12903	0.22581	0.6452
Balanced Accuracy	0.78736	0.62971	0.6068

Table 17 – PT Urban System - Decision Tree: Confusion matrix and statistics **Source:** Author, R

The representation above (**Figure 30**) shows the generated decision tree for the municipalities classification. According to the model, the split of municipalities into High/Medium/Low strongly depends on the value of the Resident Population.

The classifier made a total of 93 predictions. The correct guesses are located in the diagonal of the obtained confusion matrix (**Table 19**). Out of the 93 cases, the classifier predicted "High" 6 times, “Medium” 76 times and "Low" 11 times. In reality, the tested sample includes 12 municipalities with “High” betweenness centrality followed by 60 municipalities with “Medium” class and 21 of them have “Low” betweenness. Overall, the classifier was correct 65,59% of the time. In other words, the error rate was approximately 34,41%.

a.2. NAÏVE BAYES CLASSIFIER

The researcher selects Cross Validation as the method for error estimating. In cross-validation, the initial data is randomly partitioned into k mutually exclusive subsets, each of approximately equal size (k=10). Training and testing data is iteratively performed k times. At each iteration i, partition D_i is reserved as the test set, and the remaining partitions are collectively used to train the model.

The obtained results for the implemented Naïve Bayes model are as follows:

		PREDICTIONS		
		High	Medium	Low
CORRECT	**HIGH**	42	14	0
	MEDIUM	18	142	19
	LOW	12	55	6

ACCURACY	0.6169
95% CI	(0.5601, 0.6714)
NO INFORMATION RATE	0.6851
P-VALUE [ACC > NIR]	0.9953

KAPPA	0.2907
MCNEMAR'S TEST P-VALUE	1.371e-06

STATISTICS BY CLASS	Class: High	Class: Low	Class: Medium
SENSITIVITY	0.5833	0.24000	0.6730
SPECIFICITY	0.9407	0.76325	0.6186
POS PRED VALUE	0.7500	0.08219	0.7933
NEG PRED VALUE	0.8810	0.91915	0.4651
PREVALENCE	0.2338	0.08117	0.6851
DETECTION RATE	0.1364	0.01948	0.4610
DETECTION PREVALENCE	0.1818	0.23701	0.5812
BALANCED ACCURACY	0.7620	0.50163	0.6458

TABLE 18 – PT Urban System - Naïve Bayes: Confusion matrix and statistics **SOURCE:** Author, R

The Naïve Bayes model exhibits an overall accuracy of 61,69% for a total of 308 predictions. Out of those 308 cases, the classifier predicted "High" 72 times, “Medium” 211 times and "Low" 25 times. In reality, 56 municipalities have “High” betweenness, 179 have “Medium” and 73 have “Low” betweenness. The error rate was approximately 38,31%.

b. EUROPEAN UNION DOMAINS ANALYSIS

Given the EU-28 countries, the author collected data in *Eurostat*. To keep the consistency, data refers to the year 2011. The considered input attributes are as follows:

- Geographic Region *{Eastern Europe; Northern Europe; Southern Europe; Western Europe; Western Asia}*; Gross domestic product (GDP) at market prices, exports and imports of goods and services, current prices, million euro; Life expectancy in absolute value at birth by sex; Human Development Index (HDI); Employment and Unemployment rate (%); Resident population (No.); EU Founding father? (Y/N); Fertility rate (%); Infant Mortality Rate (%); Education (%) – up to lower secondary education (levels 0-2), from upper secondary to post-secondary non-tertiary education (levels 3 & 4) and tertiary education (levels 5-8); No. of Marriages; Crude marriage rate (%); No. of Divorces; Crude divorce Rate (%); No. of Divorces per 100 marriages.

The target attribute (eigenvector centrality) discretization is based on the following:

- "High" for the top-5 highest scores of the aggregate eigenvector outcomes (5 countries);
- "Low" for the bottom-5 lowest scores of the aggregate eigenvector outcomes (5 countries);
- "Medium", otherwise (18 countries).

b.1. DECISION TREE CLASSIFIER

The default sample percentages of 30/70 are used to partition data into test and train data, respectively. The obtained results for the decision tree predictive model are as follows:

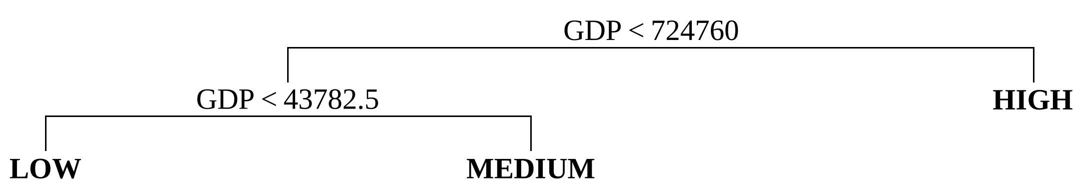


FIGURE 32 – EU Domains - Decision Tree: Representation **SOURCE:** Author, R

		Predictions		
		High	**Medium**	**Low**
Correct	**High**	0	1	0
	Medium	0	5	1
	Low	0	0	2

Accuracy	0.778
95% CI	(0.3999, 0.9719)
No Information Rate	0.6667
P-Value [Acc > NIR]	0.3772

Kappa	0.5385
Mcnemar's Test P-Value	NA

Statistics by Class	Class: High	Class: Low	Class: Medium
Sensitivity	NA	0.6667	0.8333
Specificity	0.8889	1.0000	0.6667
Pos Pred Value	NA	1.0000	0.8333
Neg Pred Value	NA	0.8571	0.6667
Prevalence	0.0000	0.3333	0.6667
Detection Rate	0.0000	0.2222	0.5556
Detection Prevalence	0.1111	0.2222	0.6667
Balanced Accuracy	NA	0.8333	0.7500

Table 19 – EU Domains - Decision Tree: Confusion matrix and statistics **Source:** Author, R

The representation above (**Figure 30**) shows the generated decision tree for the classification of the EU-28 countries. According to the model, the split of countries into High/Medium/Low strongly depends on the value of the GDP. In general:

- A GDP value higher than 724760 million $ determines a country with high eigenvector centrality;
- Countries with GDP between 724760 million $ and 43782.5 million $ are classified with a medium eigenvector centrality;
- Countries with GDP values lower than 43782.5 million $ are considered with low eigenvector centrality.

All correct guesses are located in the diagonal of the obtained confusion matrix (**Table 19**). Despite of not predicting well the class “High”, this model is shown to be more accurate when predicting the "Medium" class, with 5 countries correctly classified and only one observation wrongly predicted. The two observations classified as "Low" were well predicted. The overall accuracy of the model is then 77,8%.

b.2. NAÏVE BAYES CLASSIFIER

The obtained results for the implemented Naïve Bayes model, by using the Cross-Validation method to partition test/train data, are as follows:

		PREDICTIONS		
		HIGH	**MEDIUM**	**LOW**
CORRECT	**HIGH**	4	0	1
	MEDIUM	0	16	2
	LOW	0	0	5

ACCURACY	0.8929
95% CI	(0.7177, 0.9773)
NO INFORMATION RATE	0.6429
P-VALUE [ACC > NIR]	0.002946

KAPPA	0.8073
MCNEMAR'S TEST P-VALUE	NA

STATISTICS BY CLASS	Class: High	Class: Low	Class: Medium
SENSITIVITY	0.8000	1,0000	0.8889
SPECIFICITY	1,0000	0.8696	1,0000
POS PRED VALUE	1,0000	0.6250	1,0000
NEG PRED VALUE	0.9583	1,0000	0.8333
PREVALENCE	0.1786	0.1786	0.6429
DETECTION RATE	0.1429	0.1786	0.5714
DETECTION PREVALENCE	0.1429	0.2857	0.5714
BALANCED ACCURACY	0.9000	0.9348	0.9444

TABLE 20 – EU Domains - Naïve Bayes: Confusion matrix and statistics **SOURCE:** Author, R

The Naïve Bayes classifier exhibits a high overall accuracy of 89,29%. Among the 28 observations, the classifier correctly predicted all the "Low" cases. While for the "Medium" countries, the classifier correctly 16 out of the 18 observations, the class "High" exhibit a sensitivity 0f 80% (4 out of 5 observations were correctly classified). A Cohen's Kappa measure of around 81% indicates a good performance of the classifier. In other words, the model shows evidence that its accuracy is not simply by chance.

The following graph details the predictions executed by Naïve Bayes algorithm:

STATISTICS BY CLASS				
	Correct Class	Class: High	Class: Low	Class: Medium
[1,]	Medium	0.00%	0.00%	100.00%
[2,]	Medium	18.64%	0.00%	81.36%
[3,]	Medium	0.00%	0.00%	100.00%
[4,]	Low	0.00%	100.00%	0.00%
[5,]	Medium	0.00%	0.00%	100.00%
[6,]	High	0.00%	100.00%	0.00%
[7,]	Medium	0.00%	0.00%	100.00%
[8,]	Medium	0.00%	0.00%	100.00%
[9,]	Low	0.00%	100.00%	0.00%
[10,]	Medium	0.00%	0.00%	100.00%
[11,]	High	100.00%	0.00%	0.00%
[12,]	High	100.00%	0.00%	0.00%
[13,]	Medium	0.00%	0.00%	100.00%
[14,]	Medium	0.00%	99.63%	0.37%
[15,]	Medium	0.00%	0.00%	100.00%
[16,]	Medium	0.00%	0.00%	100.00%
[17,]	High	100.00%	0.00%	0.00%
[18,]	Low	0.00%	100.00%	0.00%
[19,]	Medium	0.00%	0.00%	100.00%
[20,]	Low	0.00%	100.00%	0.00%
[21,]	Low	0.00%	100.00%	0.00%
[22,]	High	100.00%	0.00%	0.00%
[23,]	Medium	0.00%	0.00%	100.00%
[24,]	Medium	0.00%	0.00%	100.00%
[25,]	Medium	0.00%	0.00%	100.00%
[26,]	Medium	0.00%	0.00%	100.00%
[27,]	Medium	0.00%	93.80%	6.20%
[28,]	Medium	0.00%	0.00%	100.00%

TABLE 21 – EU Domains - Naïve Bayes: Predicted data results **SOURCE:** Author, R

CHAPTER 4 – CONCLUSIONS

This chapter presents the main conclusions of the thesis and indicates how valuable these findings are. This paper also examines the theoretical and practical contributions to the field, limitations and give some suggestions for future research.

4.1. RESULTS & DISCUSSION

This paper and its performed analysis is supported by the exposure of different approaches to network measurement, distinguished in the literature.

The researcher began with networks taken from real life observations – commuting interactions due to professional and academic reasons for the study of the Portuguese municipalities and imports and exports trade, FDI and bilateral remittances interactions for the EU-28 member states case study. Each variable was initially analysed considering the main concepts and metrics cited around monoplex networks in the theoretical framework chapter. Such concepts were computed using the most suitable computer programs, namely *R*, *Muxviz*, *ArcGIS* and *Excel*, in particular to visualise the territorial networks, project maps and to carry out the overall analyses. As one would expect, some members were clearly more popular and better connected than others. The mono-layered outcomes identified Lisbon and Porto areas as the most central aggregates of Portugal while, for the European Union, several founding members (namely Germany, France and United Kingdom) were highlighted among the several mono-layered networks, though not in a totally linear way. Those conclusions were mainly explained by the concept of centrality, where the notion of "popularity" was captured (such municipalities / countries were remarked with the highest values in the computed centrality measures, including the node degree (in and out degree), betweenness and closeness), together with the flows through networks and the detected communities. This mainly constituted the univariate network analysis. With regards to the multiplex network analysis, for both case studies, individual networks were gathered and edge-coloured networks were built. A diagnostic took place to compare each individual layer with basic statistics, to examine how correlated each pair of layers were and to identify the aggregate community detection. The multiplex networks were dense and remarkably aligned to the monoplex networks – overall, members and their relationships were encountered in the different networks, creating strong basis to derive a community characterized by multi-stranded relationships. In fact, multiplex relationships may be one of the hallmarks of traditional communities. The two case studies represented large and densely connected networks. The territories are held by strong ties – the relationships between the vertices are frequent, close and intimate and, for that reason, very important. Flows through networks were critical and overall can take place through dense ties.

Furthermore, a strategic analysis was held to classify each municipality / country, according to their centrality degree. The implemented classifiers (Decision Trees and Naive Bayes.) showed higher performance for the EU domains case study.

4.2. CONCLUSION & RESEARCH IMPLICATIONS

The dissertation has regarded territorial networks of the different cities of Portugal / countries of European Union. Generally, the researcher could prove throughout the more consistent metrics and analysis and with sufficient evidence that the main characteristics of the Portuguese urban system and the European Union foundations. Taking into consideration the overall results of the research, the following conclusions can be drawn:

- For the Portuguese scenario, the results reflected the big influence of Lisbon and Porto metropolitan areas, together with discrepancies littoral/interior. Lisboa, Porto and its suburbs showed higher values of centrality degree and betweeness measures. The fraction of edges which were common to both examined layers was 63.52%, demonstrating similarities between the two considered dimensions. The archipelagos of the Azores and Madeira turned out to be more isolated from the continental territory of Portugal continent (less densely connected areas), mainly because of their geographical positioning;
- For the EU case study, one of the EU's greatest achievements of suppressing any internal borders or other regulatory obstacles to the free movement was proved. All the countries are well connected with a natural tendency for stronger connections between countries geographically closer to each other. Nevertheless, a slight greater effort from the eastern EU countries to adapt the standards of the oldest EU countries was verified. Each pair of the examined layers were higly correlated (coefficients higher than 75%) which can mean that regardless of the input data type, the EU countries show similar proximity patterns. In particular, the Imports and Exports layers, with a correlation coefficient of 0.965 showed to be very symmetric to each other: this is further proved by a reducibility of 0.078, meaning that the initial number of 4 layers could be reduced to 3 with nearly no information loss.

One of the most interesting findings is that nearly all clusters reflect geographical neighbourhoods while the input data is rather economic. Indeed, the most important factor for similarities in trade, investments and remittances seems to be the geographical distance.

The main characteristics of the networks were defined, a definition to strong links was given, there was analysed commuting links strength, there was studied the connectivity of municipalities and countries. The obtained results strongly represent the real panorama; hence, the used metrics/analysis give detailed derivations and proofs to be applied in future to modelling the behaviour of territorial networks.

4.3. Limitations & Future Research

The conclusions of the paper should consider the specificities of the case researched.

Although this study provides important insights into the analysis of mono-layered and multi-layered networks, the number of used mono-layers in the model implementation denotes one important limitation. Using only two mono-layers for the Portuguese case study (with similar connectivity behaviours) and four mono-layers for the EU study (two of them are strongly related and one has lack of data for some countries) may have biased the multiplex analysis. Therefore, this study may lack some generalizability with regard to the application of findings some trends specially in the multi-layer analysis. This simplification was employed mainly due to two reasons. First, the collected data was the one encountered in the internet, widely available. Secondly, the main goal of this research was to get conclusions from real-world datasets (no mask data was an option), so that the consistency of the outcomes and the future usability of the metrics could be verified.

The increasingly involvement of the computing intelligence and several other sciences has injected in the network analysis field a lot of new methodologies and technologies. As a result, at present, the social network analysis is a very active and hot topic in the web research. This denotes a limitation since the suggested metrics in this paper were the ones encountered in the literature to date and that seemed the most suitable ones. New features may have been tested and broadly accepted thereafter, that could allow an even more precise analysis.

Finally, the aim of the strategic analysis denotes one additional limitation. The performance of the implemented classifiers was relatively low and the accuracy was not as high as it could be expected. Specifically, the Portuguese urban system case study, where several techniques could have been examined to improve the models such as pre-processing activities, cleaning and discretization data and evaluation of the classifiers (ROC Curves). However, the researcher’s intention with the respective chapter (3.4) was mainly to go further in the analysis and not to explore the classification algorithms in depth.

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APPENDIX

1.1. PORTUGUESE URBAN SYSTEM OUTCOMES

Community detection, under clustering walktrap function, for commuting interactions due to professional reasons (**FIGURE 15**):

[1] Águeda, Aguiar da Beira, Albergaria-a-Velha, Alijó, Amarante, Amares, Anadia, Arcos de Valdevez, Arganil, Armamar, Arouca, Aveiro, Baião Barcelos, Boticas, Braga, Cabeceiras de Basto, Caminha, Carregal do Sal, Castelo de Paiva, Castro Daire, Celorico de Basto, Chaves, Cinfães, Espinho, Esposende, Estarreja, Fafe, Felgueiras, Fornos de Algodres, Gondomar, Gouveia, Guimarães, Ílhavo, Lamego, Lousada, Maia, Mangualde, Marco de Canaveses, Matosinhos, Meda, Melgaço, Mesão Frio, Mira, Moimenta da Beira, Monção, Mondim de Basto, Montalegre, Mortágua, Murça, Murtosa, Nelas, Oliveira de Azeméis, Oliveira de Frades, Oliveira do Bairro, Oliveira do Hospital, Ovar, Paços de Ferreira, Paredes, Paredes de Coura, Penafiel, Penalva do Castelo, Penedono, Peso da Régua, Ponte da Barca, Ponte de Lima, Porto, Póvoa de Lanhoso, Póvoa de Varzim, Resende, Ribeira de Pena, Sabrosa, Santa Comba Dão, Santa Maria da Feira, Santa Marta de Penaguião, Santo Tirso, São João da Madeira, São João da Pesqueira, São Pedro do Sul, Sátão, Seia, Sernancelhe, Sever do Vouga, Tábua, Tabuaço, Tarouca, Terras de Bouro, Tondela, Trofa, Vagos, Vale de Cambra, Valença, Valongo, Valpaços, Viana do Castelo, Vieira do Minho, Vila do Conde, Vila Nova de Cerveira, Vila Nova de Famalicão, Vila Nova de Foz Côa, Vila Nova de Gaia, Vila Nova de Paiva, Vila Pouca de Aguiar, Vila Real, Vila Verde, Viseu Vizela and Vouzela;

[2] Alfândega da Fé, Bragança, Carrazeda de Ansiães, Freixo de Espada à Cinta, Macedo de Cavaleiros, Miranda do Douro, Mirandela, Mogadouro, Torre de Moncorvo, Vila Flor, Vimioso and Vinhais;

[3] Aljustrel, Almodôvar, Alvito, Arraiolos, Barrancos, Beja, Castro Verde, Cuba, Évora, Ferreira do Alentejo, Mértola, Montemor-o-Novo, Mora, Moura, Mourão, Ourique, Portel, Redondo, Reguengos de Monsaraz, Serpa, Viana do Alentejo and Vidigueira;

[4] Almeida, Belmonte, Castelo Branco, Celorico da Beira, Covilhã, Figueira de Castelo, Rodrigo, Fundão, Guarda, Idanha-a-Nova, Mação, Manteigas, Oleiros, Pampilhosa da Serra, Penamacor, Pinhel, Proença-a-Nova, Sabugal, Sertã, Trancoso, Vila de Rei and Vila Velha de Ródão;

[5] Abrantes, Alcácer do Sal, Alcanena, Alcobaça, Alcochete, Alenquer, Almada, Almeirim, Alpiarça, Alvaiázere, Amadora, Ansião, Arruda dos Vinhos, Azambuja, Barreiro, Batalha, Benavente, Bombarral, Cadaval, Caldas da Rainha, Cantanhede, Cartaxo, Cascais, Castanheira de Pêra, Chamusca, Coimbra, Condeixa-a-Nova, Constância, Coruche, Entroncamento, Ferreira do Zêzere, Figueira da Foz, Figueiró dos Vinhos, Góis, Golegã, Grândola, Leiria, Lisboa, Loures, Lourinhã, Lousã, Mafra, Marinha Grande, Mealhada, Miranda do Corvo, Moita, Montemor-o-Velho, Montijo, Nazaré, Óbidos, Odemira, Odivelas, Oeiras, Ourém, Palmela, Pedrógão Grande, Penacova, Penela, Peniche, Pombal, Porto de Mós, Rio Maior, Salvaterra de Magos, Santarém, Santiago do Cacém, Sardoal, Seixal, Sesimbra, Setúbal, Sines, Sintra, Sobral de Monte Agraço, Soure, Tomar, Torres Novas, Torres Vedras, Vendas Novas, Vila Franca de Xira, Vila Nova da Barquinha, Vila Nova de Poiares;

[6] Calheta (R.A.A.) and Velas;

[7] Alandroal, Alter do Chão, Arronches, Avis, Borba, Campo Maior, Castelo de Vide Crato, Elvas, Estremoz, Fronteira, Gavião, Marvão, Monforte, Nisa, Ponte de Sor, Portalegre, Sousel and Vila Viçosa;

[8] Albufeira, Alcoutim, Aljezur, Castro Marim, Faro, Lagoa, Lagos, Loulé, Monchique, Olhão, Portimão, São Brás de Alportel, Silves, Tavira, Vila do Bispo and Vila Real de Santo António;

[9] Lagoa (R.A.A.), Nordeste, Ponta Delgada, Povoação, Ribeira Grande and Vila Franca do Campo;

[10] Lajes das Flores and Santa Cruz das Flores;

[11] Lajes do Pico, Madalena and São Roque do Pico;

[12] Calheta (R.A.M.), Câmara de Lobos, Funchal, Machico, Ponta do Sol, Porto Moniz, Ribeira Brava, Santa Cruz, Santana and São Vicente;

[13] Angra do Heroísmo and Vila da Praia da Vitória;

[14] Corvo;

[15] Horta;

[16] Porto Santo;

[17] Santa Cruz da Graciosa;

[18] Vila do Porto.

Community detection, under the clustering walktrap function, for commuting interactions due to academic reasons (**FIGURE 16**):

[1] Águeda, Aguiar da Beira, Albergaria-a-Velha, Alfândega da Fé, Alijó, Almeida, Amarante, Amares, Anadia, Arcos de Valdevez, Arganil, Armamar, Arouca, Aveiro, Baião, Barcelos, Belmonte, Boticas, Braga, Bragança, Cabeceiras de Basto, Caminha, Cantanhede, Carrazeda de Ansiães, Carregal do Sal, Castelo Branco, Castelo de Paiva, Castro Daire, Celorico da Beira, Celorico de Basto, Chaves, Cinfães, Coimbra, Condeixa-a-Nova, Covilhã, Espinho, Esposende, Estarreja, Fafe, Felgueiras, Figueira da Foz, Figueira de Castelo Rodrigo, Fornos de Algodres, Freixo de Espada à Cinta, Fundão, Góis, Gondomar, Gouveia, Guarda, Guimarães, Idanha-a-Nova, Ílhavo, Lamego, Lousã, Lousada, Macedo de Cavaleiros, Maia, Mangualde, Manteigas, Marco de Canaveses, Matosinhos, Mealhada, Meda, Melgaço, Mesão Frio, Mira, Miranda do Corvo, Miranda do Douro, Mirandela, Mogadouro, Moimenta da Beira, Monção, Mondim de Basto, Montalegre, Montemor-o-Velho, Mortágua, Murça, Murtosa, Nelas, Oliveira de Azeméis, Oliveira de Frades, Oliveira do Bairro, Oliveira do Hospital, Ovar, Paços de Ferreira, Pampilhosa da Serra, Paredes, Paredes de

Coura, Penacova, Penafiel, Penalva do Castelo, Penamacor, Penedono, Penela, Peso da Régua, Pinhel, Ponte da Barca, Ponte de Lima, Porto, Póvoa de Lanhoso, Póvoa de Varzim, Resende, Ribeira de Pena, Sabrosa, Sabugal, Santa Comba Dão, Santa Maria da Feira, Santa Marta de Penaguião, Santo Tirso, São João da Madeira, São João da Pesqueira, São Pedro do Sul, Sátão, Seia, Sernancelhe, Sever do Vouga, Soure, Tábua, Tabuaço, Tarouca, Terras de Bouro, Tondela, Torre de Moncorvo, Trancoso, Trofa, Vagos, Vale de Cambra, Valença, Valongo, Valpaços, Viana do Castelo, Vieira do Minho, Vila do Conde, Vila Flor, Vila Nova de Cerveira, Vila Nova de Famalicão, Vila Nova de Foz Côa, Vila Nova de Gaia, Vila Nova de Paiva, Vila Nova de Poiares, Vila Pouca de Aguiar, Vila Real, Vila Velha de Ródão, Vila Verde, Vimioso, Vinhais, Viseu, Vizela and Vouzela;

[2] Alcácer do Sal, Aljustrel, Almodôvar, Alvito, Barrancos, Beja, Castro Verde, Cuba, Ferreira do Alentejo Grândola, Mértola, Moura, Odemira, Ourique, Santiago do Cacém, Serpa, Sines and Vidigueira;

[3] Alandroal, Alter do Chão, Arraiolos, Arronches, Avis, Borba, Campo Maior, Castelo de Vide, Crato, Elvas, Estremoz, Évora, Fronteira, Marvão, Monforte, Montemor-o-Novo, Mora, Mourão, Nisa, Ponte de Sor, Portalegre, Portel, Redondo, Reguengos de Monsaraz, Sousel, Vendas Novas, Viana do Alentejo and Vila Viçosa;

[4] Lajes do Pico, Madalena and São Roque do Pico;

[5] Abrantes, Alcanena, Alcobaça, Alcochete, Alenquer, Almada, Almeirim, Alpiarça, Alvaiázere, Amadora, Ansião, Arruda dos Vinhos, Azambuja, Barreiro, Batalha, Benavente, Bombarral, Cadaval, Caldas da Rainha, Cartaxo, Cascais, Castanheira de Pêra, Chamusca, Constância, Coruche, Entroncamento, Ferreira do Zêzere, Figueiró dos Vinhos, Gavião, Golegã, Leiria, Lisboa, Loures, Lourinhã, Mação, Mafra, Marinha Grande, Moita, Montijo, Nazaré, Óbidos, Odivelas, Oeiras, Oleiros, Ourém, Palmela, Pedrógão Grande, Peniche, Pombal, Porto de Mós, Proença-a-Nova, Rio Maior, Salvaterra de Magos, Santarém, Sardoal, Seixal, Sertã, Sesimbra, Setúbal, Sintra, Sobral de Monte Agraço, Tomar, Torres Novas, Torres Vedras, Vila de Rei, Vila Franca de Xira and Vila Nova da Barquinha;

[6] Albufeira, Alcoutim, Aljezur, Castro Marim, Faro, Lagoa, Lagos, Loulé, Monchique, Olhão, Portimão, São Brás de Alportel, Silves, Tavira, Vila do Bispo, Vila Real de Santo António;

[7] Calheta (R.A.M.), Câmara de Lobos, Funchal, Machico, Ponta do Sol, Porto Moniz, Ribeira Brava, Santa Cruz, Santana and São Vicente;

[8] Lagoa (R.A.A.), Ponta Delgada, Ribeira Grande and Vila Franca do Campo;

NODE	MUNICIPALITY	NODE DEGREE	IN - DEGREE	OUT - DEGREE	BETWEENNESS			CLOSENESS		
					alpha=0	alpha=0.5	alpha=1	alpha=0	alpha=0.5	alpha=1
[1,]	Abrantes	160	2179	2690	610.719	3033	5338	0.00200	0.00323	0.00151
[2,]	Águeda	116	5254	3350	308.115	20	306	0.00201	0.00329	0.00164
[3,]	Aguiar da Beira	44	341	251	55.804	2	2	0.00175	0.00233	0.00088
[4,]	Alandroal	21	297	607	18.433	2	2	0.00170	0.00247	0.00119
[5,]	Albergaria-a-Velha	89	2963	3686	122.879	14	9	0.00191	0.00340	0.00167
[6,]	Albufeira	140	6156	2204	320.057	341	2074	0.00184	0.00263	0.00127
[7,]	Alcácer do Sal	64	524	892	79.309	0	274	0.00181	0.00275	0.00126
[8,]	Alcanena	77	2478	1401	53.899	10	0	0.00182	0.00308	0.00154
[9,]	Alcobaça	88	4013	4856	184.162	1058	2430	0.00198	0.00338	0.00165
[10,]	Alcochete	84	3648	4556	105.557	298	0	0.00187	0.00357	0.00158
[11,]	Alcoutim	26	124	100	9.449	2	0	0.00162	0.00176	0.00053
[12,]	Alenquer	135	4877	6790	359.429	13	0	0.00197	0.00358	0.00159
[13,]	Alfândega da Fé	46	186	184	20.898	0	0	0.00168	0.00170	0.00063
[14,]	Alijó	67	627	407	65.787	26	378	0.00177	0.00260	0.00131
[15,]	Aljezur	27	235	265	11.652	45	54	0.00166	0.00202	0.00098
[16,]	Aljustrel	64	442	836	51.128	30	43	0.00175	0.00225	0.00098
[17,]	Almada	185	18964	35003	1640.474	593	2342	0.00221	0.00378	0.00161
[18,]	Almeida	91	959	247	166.001	1326	2642	0.00173	0.00227	0.00100
[19,]	Almeirim	69	1530	2906	89.911	19	319	0.00191	0.00328	0.00158
[20,]	Almodôvar	61	347	700	42.091	34	44	0.00169	0.00197	0.00084
[21,]	Alpiarça	35	456	1240	23.905	0	0	0.00181	0.00304	0.00151

Node	Municipality	Node	In -	Out -	Betweenness			Closeness		
[22,]	Alter do Chão	39	269	183	18.342	0	7	0.00165	0.00189	0.00071
[23,]	Alvaiázere	43	526	592	31.996	6	0	0.00177	0.00264	0.00129
[24,]	Alvito	27	163	255	12.371	0	0	0.00166	0.00233	0.00106
[25,]	Amadora	158	25249	46306	971.674	0	0	0.00222	0.00379	0.00160
[26,]	Amarante	101	3093	5140	481.276	657	7119	0.00210	0.00362	0.00173
[27,]	Amares	51	1175	2990	95.773	0	0	0.00188	0.00344	0.00172
[28,]	Anadia	74	2578	3420	146.006	11	98	0.00193	0.00346	0.00167
[29,]	Angra do Heroísmo	54	2057	868	260.930	667	914	0.00171	0.00187	0.00072
[30,]	Ansião	49	914	1296	75.290	733	2161	0.00188	0.00321	0.00160
[31,]	Arcos de Valdevez	47	1502	957	40.526	6	305	0.00180	0.00262	0.00142
[32,]	Arganil	62	797	607	74.684	8	273	0.00181	0.00275	0.00121
[33,]	Armamar	43	364	270	46.592	0	0	0.00173	0.00236	0.00104
[34,]	Arouca	64	1213	2435	78.374	0	0	0.00187	0.00322	0.00163
[35,]	Arraiolos	38	402	919	46.183	0	0	0.00178	0.00279	0.00137
[36,]	Arronches	17	129	238	12.168	0	0	0.00166	0.00205	0.00082
[37,]	Arruda dos Vinhos	56	1448	3310	38.822	0	0	0.00181	0.00345	0.00156
[38,]	Aveiro	186	17398	6761	1113.890	8125	20100	0.00219	0.00371	0.00170
[39,]	Avis	46	325	181	39.266	22	21	0.00171	0.00206	0.00089
[40,]	Azambuja	104	5389	3199	167.251	269	338	0.00188	0.00346	0.00157
[41,]	Baião	67	694	1477	166.914	395	501	0.00201	0.00332	0.00160
[42,]	Barcelos	90	7919	8837	377.596	1829	3868	0.00212	0.00350	0.00172
[43,]	Barrancos	32	72	44	5.455	0	0	0.00157	0.00153	0.00041
[44,]	Barreiro	108	7655	17190	408.683	296	298	0.00204	0.00373	0.00160
[45,]	Batalha	100	2907	2696	129.795	0	0	0.00187	0.00331	0.00165
[46,]	Beja	159	2677	1330	505.199	4948	5542	0.00187	0.00267	0.00107
[47,]	Belmonte	33	843	604	74.236	0	0	0.00175	0.00252	0.00117
[48,]	Benavente	112	3310	3933	282.759	302	857	0.00196	0.00351	0.00157
[49,]	Bombarral	51	1103	1559	20.763	0	1	0.00177	0.00308	0.00152
[50,]	Borba	35	538	813	55.218	0	0	0.00176	0.00229	0.00120
[51,]	Boticas	27	219	194	7.644	0	0	0.00168	0.00207	0.00105
[52,]	Braga	157	17719	13590	1625.274	7454	4564	0.00234	0.00374	0.00175
[53,]	Bragança	137	1244	1003	354.612	2229	2993	0.00188	0.00235	0.00100

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[54,]	Cabeceiras de Basto	44	542	864	40.328	248	29	0.00188	0.00286	0.00134
[55,]	Cadaval	55	856	1749	69.024	0	0	0.00177	0.00316	0.00153
[56,]	Caldas da Rainha	116	4682	4629	277.519	1209	2296	0.00200	0.00345	0.00164
[57,]	Calheta (R.A.A.)	43	316	105	35.811	321	340	0.00126	0.00099	0.00010
[58,]	Calheta (R.A.M.)	13	286	697	17.862	0	0	0.00155	0.00223	0.00096
[59,]	Câmara de Lobos	16	2269	7009	63.341	11	0	0.00171	0.00249	0.00102
[60,]	Caminha	31	826	1584	65.524	1	1204	0.00181	0.00308	0.00162
[61,]	Campo Maior	47	438	269	44.999	0	0	0.00163	0.00223	0.00100
[62,]	Cantanhede	80	3619	4122	189.966	419	298	0.00204	0.00373	0.00174
[63,]	Carrazeda de Ansiães	48	238	146	29.468	2	1	0.00171	0.00164	0.00067
[64,]	Carregal do Sal	53	711	801	75.390	2	0	0.00185	0.00290	0.00136
[65,]	Cartaxo	84	1520	4374	144.400	0	108	0.00193	0.00347	0.00159
[66,]	Cascais	171	18689	39174	1223.268	2	0	0.00226	0.00377	0.00160
[67,]	Castanheira de Pêra	43	230	167	23.825	0	0	0.00166	0.00206	0.00079
[68,]	Castelo Branco	160	1827	1853	968.848	3206	5948	0.00207	0.00307	0.00135
[69,]	Castelo de Paiva	55	833	1875	80.708	195	0	0.00192	0.00326	0.00159
[70,]	Castelo de Vide	22	204	213	24.214	0	0	0.00168	0.00202	0.00081
[71,]	Castro Daire	52	551	577	97.771	0	0	0.00189	0.00295	0.00138
[72,]	Castro Marim	26	561	902	17.547	164	1016	0.00168	0.00259	0.00127
[73,]	Castro Verde	70	1289	381	58.197	337	896	0.00170	0.00205	0.00086
[74,]	Celorico da Beira	35	415	624	46.295	1	28	0.00176	0.00252	0.00114
[75,]	Celorico de Basto	52	732	1860	84.514	492	575	0.00193	0.00315	0.00155
[76,]	Chamusca	55	640	1193	41.221	0	0	0.00180	0.00290	0.00145
[77,]	Chaves	84	883	1146	183.963	1103	1319	0.00193	0.00262	0.00130
[78,]	Cinfães	59	585	1181	171.530	1	0	0.00199	0.00317	0.00152
[79,]	Coimbra	217	22189	8652	2391.937	16039	24345	0.00240	0.00412	0.00178
[80,]	Condeixa-a-Nova	73	1267	4145	214.609	20	0	0.00200	0.00382	0.00176
[81,]	Constância	118	1382	590	99.069	0	0	0.00174	0.00270	0.00137
[82,]	Coruche	84	906	1588	121.247	8	4	0.00190	0.00316	0.00146
[83,]	Corvo	10	15	0	0.000	0	0	0.00000	0.00000	0.00000
[84,]	Covilhã	114	1852	2614	490.426	1755	4613	0.00202	0.00292	0.00128
[85,]	Crato	23	167	299	20.972	0	0	0.00168	0.00205	0.00082

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[86,]	Cuba	25	216	644	25.144	0	0	0.00174	0.00235	0.00102
[87,]	Elvas	98	736	926	198.133	609	777	0.00182	0.00272	0.00113
[88,]	Entroncamento	130	3003	4424	316.651	587	3024	0.00196	0.00348	0.00165
[89,]	Espinho	113	4415	5655	391.438	9	0	0.00198	0.00370	0.00173
[90,]	Esposende	58	2389	4386	97.463	28	0	0.00199	0.00339	0.00169
[91,]	Estarreja	84	2997	4233	152.080	312	610	0.00193	0.00339	0.00169
[92,]	Estremoz	93	1080	1045	288.252	1569	3513	0.00187	0.00267	0.00131
[93,]	Évora	157	5010	1899	658.587	11051	12783	0.00198	0.00314	0.00142
[94,]	Fafe	67	2010	4218	120.667	802	918	0.00194	0.00332	0.00170
[95,]	Faro	153	10387	5042	511.797	4705	8774	0.00191	0.00301	0.00133
[96,]	Felgueiras	68	6432	3627	180.900	192	6908	0.00202	0.00338	0.00166
[97,]	Ferreira do Alentejo	57	515	557	65.578	14	0	0.00177	0.00228	0.00099
[98,]	Ferreira do Zêzere	51	560	638	43.286	18	158	0.00177	0.00280	0.00145
[99,]	Figueira da Foz	154	3896	3944	642.221	176	3891	0.00215	0.00373	0.00173
[100,]	Figueira de Castelo Rodrigo	35	132	154	42.877	120	0	0.00169	0.00175	0.00070
[101,]	Figueiró dos Vinhos	41	362	576	38.513	280	1130	0.00179	0.00258	0.00136
[102,]	Fornos de Algodres	36	329	350	30.696	1	0	0.00173	0.00247	0.00101
[103,]	Freixo de Espada à Cinta	33	122	61	11.773	2	0	0.00156	0.00141	0.00043
[104,]	Fronteira	41	209	270	71.199	22	1	0.00172	0.00185	0.00070
[105,]	Funchal	88	20044	4633	600.048	6918	6872	0.00181	0.00257	0.00103
[106,]	Fundão	79	1575	1681	282.390	222	287	0.00190	0.00278	0.00129
[107,]	Gavião	31	170	221	39.403	1	0	0.00171	0.00223	0.00096
[108,]	Góis	37	260	230	38.062	1	0	0.00173	0.00244	0.00105
[109,]	Golegã	40	509	906	9.191	0	0	0.00176	0.00281	0.00146
[110,]	Gondomar	97	8137	38097	647.034	65	0	0.00234	0.00389	0.00176
[111,]	Gouveia	53	548	784	58.092	8	3	0.00176	0.00257	0.00121
[112,]	Grândola	82	1005	1012	64.129	14	315	0.00176	0.00252	0.00107
[113,]	Guarda	148	2320	1828	769.940	4550	8249	0.00202	0.00293	0.00121
[114,]	Guimarães	117	13615	11090	514.181	2331	8689	0.00215	0.00357	0.00173
[115,]	Horta	25	74	64	98.450	152	866	0.00159	0.00112	0.00018
[116,]	Idanha-a-Nova	36	432	231	23.736	0	0	0.00169	0.00244	0.00115
[117,]	Ílhavo	67	3341	6313	147.174	0	516	0.00206	0.00350	0.00169

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[118,]	Lagoa	76	3521	3357	170.109	2197	3222	0.00178	0.00262	0.00121
[119,]	Lagoa (R.A.A.)	0	0	2683	0.000	0	0	0.00000	0.00000	0.00000
[120,]	Lagos	70	1511	1614	74.824	1202	971	0.00177	0.00251	0.00119
[121,]	Lajes das Flores	12	35	86	10.254	2	310	0.00120	0.00133	0.00030
[122,]	Lajes do Pico	16	110	249	13.449	206	20	0.00142	0.00105	0.00016
[123,]	Lamego	105	1415	1915	341.266	2293	2979	0.00205	0.00331	0.00145
[124,]	Leiria	197	11835	8900	1456.396	4641	8593	0.00223	0.00363	0.00169
[125,]	Lisboa	298	324121	37097	6635.304	56016	53188	0.00253	0.00387	0.00161
[126,]	Loulé	110	6121	4279	251.103	864	4562	0.00188	0.00283	0.00131
[127,]	Loures	162	29319	49660	1093.088	3	2341	0.00230	0.00380	0.00161
[128,]	Lourinhã	63	1310	3055	93.033	268	1638	0.00187	0.00331	0.00158
[129,]	Lousã	78	1030	2525	153.143	1	600	0.00193	0.00372	0.00174
[130,]	Lousada	89	4246	7007	323.297	21	114	0.00204	0.00355	0.00172
[131,]	Mação	57	411	324	71.712	17	289	0.00175	0.00243	0.00115
[132,]	Macedo de Cavaleiros	71	554	621	110.543	1	299	0.00180	0.00220	0.00100
[133,]	Machico	17	1379	2853	77.124	0	305	0.00172	0.00241	0.00101
[134,]	Madalena	33	332	228	20.998	971	1509	0.00150	0.00095	0.00016
[135,]	Mafra	139	5451	15422	451.160	300	2739	0.00203	0.00370	0.00160
[136,]	Maia	143	31077	30481	979.645	559	14040	0.00229	0.00384	0.00176
[137,]	Mangualde	83	2381	1544	262.609	6	585	0.00200	0.00332	0.00158
[138,]	Manteigas	30	122	139	7.353	0	0	0.00166	0.00208	0.00084
[139,]	Marco de Canaveses	80	2591	3953	332.861	756	1144	0.00215	0.00361	0.00172
[140,]	Marinha Grande	81	3981	3606	126.442	0	0	0.00192	0.00339	0.00167
[141,]	Marvão	24	176	317	15.583	0	0	0.00166	0.00215	0.00087
[142,]	Matosinhos	139	29277	33091	1147.790	571	577	0.00242	0.00387	0.00176
[143,]	Mealhada	79	2241	3766	140.323	0	293	0.00195	0.00376	0.00175
[144,]	Meda	49	208	215	37.967	126	536	0.00168	0.00183	0.00073
[145,]	Melgaço	28	293	257	7.270	0	0	0.00169	0.00195	0.00096
[146,]	Mértola	50	257	281	37.232	42	20	0.00169	0.00204	0.00085
[147,]	Mesão Frio	32	316	353	19.385	0	0	0.00176	0.00239	0.00121
[148,]	Mira	41	763	1371	66.549	0	0	0.00188	0.00319	0.00160
[149,]	Miranda do Corvo	37	650	2797	50.460	0	0	0.00187	0.00377	0.00175

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[150,]	Miranda do Douro	48	247	131	39.292	0	0	0.00170	0.00167	0.00053
[151,]	Mirandela	92	981	800	189.252	1620	3850	0.00186	0.00253	0.00113
[152,]	Mogadouro	67	254	240	76.229	14	5	0.00175	0.00171	0.00058
[153,]	Moimenta da Beira	49	487	449	71.918	97	971	0.00179	0.00247	0.00098
[154,]	Moita	77	3558	15902	206.426	0	0	0.00202	0.00370	0.00160
[155,]	Monção	52	578	910	62.596	308	610	0.00175	0.00237	0.00116
[156,]	Monchique	22	270	501	6.788	0	0	0.00163	0.00238	0.00114
[157,]	Mondim de Basto	33	381	400	13.218	1	0	0.00176	0.00250	0.00125
[158,]	Monforte	31	246	226	24.123	0	0	0.00164	0.00196	0.00077
[159,]	Montalegre	57	417	229	49.546	7	0	0.00174	0.00228	0.00089
[160,]	Montemor-o-Novo	82	804	1387	155.023	46	65	0.00186	0.00286	0.00136
[161,]	Montemor-o-Velho	67	1367	5491	230.130	0	0	0.00206	0.00383	0.00176
[162,]	Montijo	112	6464	10603	321.720	549	836	0.00200	0.00368	0.00160
[163,]	Mora	51	235	292	49.981	0	0	0.00167	0.00250	0.00097
[164,]	Mortágua	53	645	699	51.809	1	1	0.00181	0.00292	0.00133
[165,]	Moura	63	502	418	109.703	197	611	0.00177	0.00208	0.00090
[166,]	Mourão	31	129	130	14.762	0	0	0.00159	0.00212	0.00088
[167,]	Murça	41	258	262	24.092	0	0	0.00174	0.00241	0.00115
[168,]	Murtosa	43	746	1476	46.719	0	0	0.00182	0.00309	0.00160
[169,]	Nazaré	70	1110	1652	81.665	0	0	0.00182	0.00303	0.00159
[170,]	Nelas	59	1167	1391	339.259	192	631	0.00192	0.00317	0.00152
[171,]	Nisa	39	300	401	45.996	28	34	0.00173	0.00218	0.00088
[172,]	Nordeste	12	200	176	9.650	0	0	0.00139	0.00191	0.00096
[173,]	Óbidos	64	1948	1965	47.844	0	0	0.00183	0.00313	0.00160
[174,]	Odemira	78	528	1199	110.595	101	611	0.00178	0.00231	0.00104
[175,]	Odivelas	104	10040	42630	703.307	0	0	0.00224	0.00378	0.00160
[176,]	Oeiras	198	49949	41713	1274.031	0	302	0.00223	0.00378	0.00160
[177,]	Oleiros	58	336	203	34.832	2	0	0.00170	0.00220	0.00094
[178,]	Olhão	62	1789	6765	152.906	0	1956	0.00186	0.00292	0.00132
[179,]	Oliveira de Azeméis	106	7625	8102	428.262	729	1367	0.00201	0.00350	0.00170
[180,]	Oliveira de Frades	86	2038	866	72.773	151	268	0.00183	0.00274	0.00138
[181,]	Oliveira do Bairro	56	3310	3584	84.303	85	391	0.00195	0.00335	0.00166

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[182,]	Oliveira do Hospital	76	1132	1254	136.970	246	671	0.00189	0.00295	0.00133
[183,]	Ourém	142	3596	3213	332.552	1256	4373	0.00196	0.00334	0.00166
[184,]	Ourique	51	346	312	36.310	4	5	0.00171	0.00184	0.00072
[185,]	Ovar	104	6527	7198	384.033	626	810	0.00213	0.00363	0.00172
[186,]	Paços de Ferreira	88	5755	4782	185.355	12	38	0.00190	0.00341	0.00172
[187,]	Palmela	123	13282	12295	365.054	43	922	0.00201	0.00366	0.00160
[188,]	Pampilhosa da Serra	56	267	173	33.183	3	0	0.00170	0.00216	0.00095
[189,]	Paredes	100	7510	12097	390.355	838	2353	0.00213	0.00374	0.00175
[190,]	Paredes de Coura	33	339	904	18.166	0	0	0.00173	0.00248	0.00128
[191,]	Pedrógão Grande	42	277	257	44.546	23	38	0.00173	0.00197	0.00096
[192,]	Penacova	38	632	2851	60.451	0	0	0.00190	0.00377	0.00175
[193,]	Penafiel	95	6666	8644	602.828	1426	1235	0.00226	0.00378	0.00175
[194,]	Penalva do Castelo	35	377	944	43.571	0	0	0.00180	0.00308	0.00148
[195,]	Penamacor	21	224	140	10.113	0	0	0.00164	0.00195	0.00072
[196,]	Penedono	27	146	142	24.307	3	4	0.00168	0.00156	0.00052
[197,]	Penela	46	574	709	46.473	0	0	0.00180	0.00331	0.00160
[198,]	Peniche	76	1233	1702	146.477	2	1159	0.00186	0.00312	0.00151
[199,]	Peso da Régua	81	1625	1217	120.869	861	2553	0.00185	0.00290	0.00147
[200,]	Pinhel	45	323	550	50.333	0	0	0.00176	0.00247	0.00111
[201,]	Pombal	117	4198	4157	375.670	811	4664	0.00208	0.00360	0.00171
[202,]	Ponta Delgada	75	6263	2496	792.008	3445	5634	0.00179	0.00249	0.00128
[203,]	Ponta do Sol	17	578	921	24.251	0	0	0.00164	0.00228	0.00098
[204,]	Ponte da Barca	32	692	1147	50.796	0	0	0.00177	0.00264	0.00137
[205,]	Ponte de Lima	58	2132	3673	153.007	486	1814	0.00196	0.00321	0.00165
[206,]	Ponte de Sor	85	602	676	186.556	913	2542	0.00187	0.00278	0.00121
[207,]	Portalegre	148	1585	945	494.283	4068	3600	0.00185	0.00255	0.00095
[208,]	Portel	83	507	665	105.187	323	601	0.00175	0.00270	0.00132
[209,]	Portimão	128	4779	3725	406.381	4727	5511	0.00187	0.00276	0.00123
[210,]	Porto	234	113763	23393	2821.184	33244	43776	0.00236	0.00397	0.00176
[211,]	Porto de Mós	61	1822	3198	103.451	23	46	0.00189	0.00323	0.00163
[212,]	Porto Moniz	26	170	124	31.582	0	0	0.00150	0.00188	0.00075
[213,]	Porto Santo	20	44	13	6.881	0	0	0.00147	0.00141	0.00035

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[214,]	Póvoa de Lanhoso	47	1671	2232	35.636	0	0	0.00185	0.00336	0.00170
[215,]	Póvoa de Varzim	100	6574	7548	315.900	13	17	0.00200	0.00359	0.00172
[216,]	Povoação	13	180	186	10.235	0	0	0.00143	0.00194	0.00099
[217,]	Proença-a-Nova	28	221	471	25.683	0	252	0.00172	0.00240	0.00111
[218,]	Redondo	37	294	629	49.470	0	0	0.00174	0.00271	0.00133
[219,]	Reguengos de Monsaraz	55	522	677	60.342	304	303	0.00175	0.00272	0.00134
[220,]	Resende	42	332	639	60.415	0	0	0.00187	0.00273	0.00115
[221,]	Ribeira Brava	13	1019	1980	22.913	77	1	0.00166	0.00240	0.00101
[222,]	Ribeira de Pena	44	312	234	57.507	1	0	0.00173	0.00207	0.00089
[223,]	Ribeira Grande	13	2000	3660	131.900	908	913	0.00172	0.00238	0.00127
[224,]	Rio Maior	80	1743	1844	90.076	72	62	0.00189	0.00317	0.00157
[225,]	Sabrosa	45	412	483	42.741	0	0	0.00175	0.00277	0.00143
[226,]	Sabugal	37	383	360	34.261	34	0	0.00173	0.00239	0.00107
[227,]	Salvaterra de Magos	77	997	3043	131.333	2	0	0.00189	0.00338	0.00155
[228,]	Santa Comba Dão	54	663	1300	96.011	10	147	0.00188	0.00308	0.00150
[229,]	Santa Cruz	36	3535	11131	325.515	2749	4686	0.00176	0.00251	0.00102
[230,]	Santa Cruz da Graciosa	5	10	15	3.474	0	0	0.00152	0.00139	0.00035
[231,]	Santa Cruz das Flores	8	87	27	16.125	7	0	0.00157	0.00111	0.00023
[232,]	Santa Maria da Feira	134	10905	17770	807.479	2402	4431	0.00227	0.00383	0.00175
[233,]	Santa Marta de Penaguião	27	312	753	22.537	0	0	0.00180	0.00280	0.00144
[234,]	Santana	60	409	512	123.224	0	0	0.00156	0.00223	0.00096
[235,]	Santarém	170	7087	5160	583.421	3233	5633	0.00206	0.00356	0.00162
[236,]	Santiago do Cacém	81	1166	3739	129.657	623	1448	0.00184	0.00269	0.00113
[237,]	Santo Tirso	77	7660	9153	193.891	50	69	0.00202	0.00358	0.00173
[238,]	São Brás de Alportel	25	557	1740	23.261	0	0	0.00174	0.00272	0.00129
[239,]	São João da Madeira	91	9733	3606	135.638	0	0	0.00185	0.00345	0.00171
[240,]	São João da Pesqueira	49	313	228	60.424	82	5	0.00178	0.00183	0.00050
[241,]	São Pedro do Sul	58	663	1206	92.574	0	0	0.00188	0.00308	0.00147
[242,]	São Roque do Pico	16	285	170	33.301	16	0	0.00159	0.00090	0.00016
[243,]	São Vicente	21	433	304	26.853	0	0	0.00159	0.00214	0.00092
[244,]	Sardoal	25	280	509	17.611	0	0	0.00172	0.00272	0.00139
[245,]	Sátão	33	461	1201	56.953	0	296	0.00189	0.00331	0.00158

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[246,]	Seia	71	1037	1254	187.864	144	681	0.00193	0.00284	0.00129
[247,]	Seixal	120	9130	40113	590.620	298	2053	0.00219	0.00378	0.00161
[248,]	Sernancelhe	26	285	346	27.484	267	580	0.00173	0.00220	0.00081
[249,]	Serpa	55	372	700	124.484	0	608	0.00177	0.00233	0.00102
[250,]	Sertã	66	551	635	135.664	290	157	0.00183	0.00241	0.00090
[251,]	Sesimbra	65	2401	10580	152.591	0	0	0.00199	0.00367	0.00160
[252,]	Setúbal	171	11764	14530	928.414	1699	2556	0.00214	0.00369	0.00160
[253,]	Sever do Vouga	63	691	1142	31.298	9	9	0.00178	0.00289	0.00148
[254,]	Silves	65	1889	5378	217.681	29	355	0.00186	0.00256	0.00126
[255,]	Sines	151	5521	440	131.393	577	1101	0.00173	0.00236	0.00105
[256,]	Sintra	210	27377	86694	2706.789	1947	7340	0.00249	0.00381	0.00161
[257,]	Sobral de Monte Agraço	35	905	2428	19.230	0	0	0.00179	0.00335	0.00153
[258,]	Soure	29	865	3441	56.268	0	0	0.00196	0.00372	0.00174
[259,]	Sousel	41	249	318	46.278	8	300	0.00172	0.00216	0.00110
[260,]	Tábua	63	1101	1000	119.735	10	68	0.00180	0.00282	0.00126
[261,]	Tabuaço	39	222	266	43.247	0	296	0.00175	0.00211	0.00078
[262,]	Tarouca	45	450	615	67.854	0	0	0.00181	0.00278	0.00133
[263,]	Tavira	72	1419	2068	172.607	1029	1979	0.00182	0.00284	0.00130
[264,]	Terras de Bouro	27	377	483	5.665	0	0	0.00174	0.00298	0.00152
[265,]	Tomar	138	2509	2938	417.281	954	3556	0.00202	0.00336	0.00161
[266,]	Tondela	92	1683	1885	291.923	100	763	0.00200	0.00338	0.00159
[267,]	Torre de Moncorvo	88	414	227	119.653	530	527	0.00174	0.00189	0.00058
[268,]	Torres Novas	162	4557	4590	507.432	1237	4334	0.00198	0.00339	0.00162
[269,]	Torres Vedras	113	5281	7629	310.104	1098	2716	0.00204	0.00362	0.00160
[270,]	Trancoso	72	560	429	116.467	374	1115	0.00179	0.00239	0.00096
[271,]	Trofa	94	7174	6624	227.309	0	10948	0.00192	0.00356	0.00174
[272,]	Vagos	95	1619	3835	214.809	94	214	0.00192	0.00336	0.00166
[273,]	Vale de Cambra	70	2350	1960	71.069	12	243	0.00183	0.00313	0.00164
[274,]	Valença	54	1610	1091	32.845	304	1216	0.00174	0.00248	0.00123
[275,]	Valongo	99	8305	22672	424.535	4	2357	0.00218	0.00382	0.00176
[276,]	Valpaços	37	389	435	52.413	31	54	0.00181	0.00222	0.00115
[277,]	Velas	24	136	108	65.933	132	15	0.00164	0.00104	0.00010

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[278,]	Vendas Novas	94	1047	1199	182.198	13	12	0.00183	0.00300	0.00142
[279,]	Viana do Alentejo	44	278	726	49.549	0	292	0.00175	0.00276	0.00135
[280,]	Viana do Castelo	116	5993	4877	492.487	3211	5178	0.00205	0.00347	0.00169
[281,]	Vidigueira	25	273	550	22.944	0	0	0.00171	0.00232	0.00101
[282,]	Vieira do Minho	51	617	814	85.394	0	271	0.00188	0.00296	0.00151
[283,]	Vila da Praia da Vitória	7	789	1996	11.639	2	308	0.00163	0.00179	0.00072
[284,]	Vila de Rei	31	199	158	14.625	0	0	0.00164	0.00195	0.00078
[285,]	Vila do Bispo	33	511	279	6.604	0	0	0.00158	0.00213	0.00107
[286,]	Vila do Conde	104	9782	11670	420.027	292	567	0.00212	0.00367	0.00174
[287,]	Vila do Porto	12	26	11	3.261	0	0	0.00143	0.00136	0.00022
[288,]	Vila Flor	58	436	297	46.159	65	910	0.00171	0.00204	0.00095
[289,]	Vila Franca de Xira	161	12371	34707	1205.275	2037	2070	0.00220	0.00378	0.00161
[290,]	Vila Franca do Campo	16	372	943	27.896	0	0	0.00158	0.00226	0.00124
[291,]	Vila Nova da Barquinha	89	820	1616	157.242	0	0	0.00183	0.00302	0.00155
[292,]	Vila Nova de Cerveira	44	1714	753	24.486	2	1815	0.00173	0.00257	0.00127
[293,]	Vila Nova de Famalicão	119	12333	13706	531.035	156	11880	0.00218	0.00366	0.00174
[294,]	Vila Nova de Foz Côa	72	350	178	56.853	86	64	0.00171	0.00158	0.00054
[295,]	Vila Nova de Gaia	156	22309	46708	1467.016	13200	30121	0.00254	0.00407	0.00177
[296,]	Vila Nova de Paiva	38	331	284	21.834	0	0	0.00171	0.00279	0.00129
[297,]	Vila Nova de Poiares	41	648	906	26.365	0	0	0.00175	0.00346	0.00166
[298,]	Vila Pouca de Aguiar	49	557	557	41.038	2	279	0.00178	0.00266	0.00136
[299,]	Vila Real	158	3766	2625	952.895	8516	15520	0.00211	0.00332	0.00158
[300,]	Vila Real de Santo António	49	1149	1118	37.530	134	1566	0.00173	0.00259	0.00125
[301,]	Vila Velha de Ródão	23	383	193	10.004	0	0	0.00163	0.00244	0.00115
[302,]	Vila Verde	49	2715	6114	90.873	295	0	0.00198	0.00353	0.00173
[303,]	Vila Viçosa	41	806	817	78.267	23	857	0.00178	0.00244	0.00116
[304,]	Vimioso	13	134	146	1.603	0	0	0.00165	0.00185	0.00081
[305,]	Vinhais	21	184	260	9.909	0	0	0.00171	0.00197	0.00089
[306,]	Viseu	177	5511	5643	1262.665	11723	13621	0.00230	0.00383	0.00165
[307,]	Vizela	63	2757	3759	65.257	0	0	0.00186	0.00332	0.00170
[308,]	Vouzela	55	1241	1344	45.465	0	344	0.00183	0.00313	0.00150

Table 22 – Outcomes for each Portuguese municipality, commuting interactions due to professional reasons variable, 2011: Node Degree, In-Degree, Out-Degree, and Betweenness and Closeness, considering different levels of alpha

NODE	MUNICIPALITY	NODE DEGREE	IN - DEGREE	OUT - DEGREE	BETWEENNESS			CLOSENESS		
					alpha=0	alpha=0.5	alpha=1	alpha=0	alpha=0.5	alpha=1
[1,]	Abrantes	72	426	633	1516.799	2678	5498	0.00161	0.00162	5.28E-04
[2,]	Águeda	51	750	998	406.572	531	2290	0.00169	0.00165	5.48E-04
[3,]	Aguiar da Beira	7	39	116	52.956	1	0	0.00149	0.00142	4.81E-04
[4,]	Alandroal	6	18	184	43.315	16	2	0.00139	0.00125	4.56E-04
[5,]	Albergaria-a-Velha	17	398	655	66.228	34	5	0.00161	0.00157	5.44E-04
[6,]	Albufeira	22	504	710	174.154	78	2864	0.00151	0.00147	5.27E-04
[7,]	Alcácer do Sal	13	34	184	65.041	2	2	0.00144	0.00139	4.81E-04
[8,]	Alcanena	18	192	404	24.421	39	4936	0.00151	0.00153	5.24E-04
[9,]	Alcobaça	21	631	1387	200.299	1080	1260	0.00168	0.00173	5.48E-04
[10,]	Alcochete	17	454	849	90.431	12	0	0.00158	0.00164	5.35E-04
[11,]	Alcoutim	12	44	57	89.578	31	306	0.00134	0.00112	4.14E-04
[12,]	Alenquer	19	212	1532	110.983	295	290	0.00167	0.00168	5.39E-04
[13,]	Alfândega da Fé	4	4	57	31.701	0	0	0.00139	0.00117	4.18E-04
[14,]	Alijó	16	49	205	241.395	41	343	0.00148	0.00139	5.01E-04
[15,]	Aljezur	9	27	141	55.434	6	0	0.00139	0.00131	4.79E-04
[16,]	Aljustrel	6	16	180	15.592	5	0	0.00147	0.00134	4.85E-04
[17,]	Almada	142	7193	4717	3274.149	6814	7805	0.00175	0.00173	5.42E-04
[18,]	Almeida	31	175	116	300.204	1352	3122	0.00154	0.00134	4.51E-04
[19,]	Almeirim	15	253	505	218.965	100	1523	0.00161	0.00156	5.34E-04
[20,]	Almodôvar	8	26	64	17.072	42	329	0.00136	0.00118	4.11E-04
[21,]	Alpiarça	3	32	327	22.637	0	0	0.00153	0.00149	5.21E-04
[22,]	Alter do Chão	28	60	72	182.348	42	11	0.00134	0.00115	3.95E-04
[23,]	Alvaiázere	12	52	189	38.981	28	0	0.00145	0.00142	4.97E-04
[24,]	Alvito	19	133	74	44.542	15	278	0.00127	0.00118	4.22E-04
[25,]	Amadora	52	3861	8817	874.47	388	0	0.00175	0.00174	5.42E-04
[26,]	Amarante	30	1336	1297	359.953	1210	5713	0.00164	0.00172	5.51E-04
[27,]	Amares	19	148	879	51.56	0	0	0.00161	0.00169	5.51E-04
[28,]	Anadia	20	631	853	76.288	22	145	0.00159	0.00162	5.50E-04

Node	Municipality	Node	In -	Out -	Betweenness			Closeness		
[29,]	Angra do Heroísmo	30	416	322	505.358	591	690	0.00162	0.00152	5.09E-04
[30,]	Ansião	13	265	267	206.684	437	2235	0.00155	0.00153	5.18E-04
[31,]	Arcos de Valdevez	13	203	266	40.773	3	313	0.00152	0.00143	4.84E-04
[32,]	Arganil	16	186	138	85.478	26	589	0.00147	0.00145	5.04E-04
[33,]	Armamar	7	24	281	52.693	31	336	0.00150	0.00144	5.02E-04
[34,]	Arouca	11	61	466	11.777	0	0	0.00156	0.00155	5.19E-04
[35,]	Arraiolos	7	42	239	22.83	603	1583	0.00134	0.00144	5.12E-04
[36,]	Arronches	7	14	106	6.71	0	0	0.00126	0.00125	4.37E-04
[37,]	Arruda dos Vinhos	15	714	499	7.43	15	0	0.00143	0.00162	5.31E-04
[38,]	Aveiro	198	5424	815	3008.026	2836	6287	0.00170	0.00170	5.53E-04
[39,]	Avis	17	43	103	113.526	32	5	0.00143	0.00123	4.26E-04
[40,]	Azambuja	20	225	663	30.693	7	0	0.00156	0.00163	5.35E-04
[41,]	Baião	12	63	530	80.491	404	585	0.00160	0.00155	5.20E-04
[42,]	Barcelos	61	1646	3422	642.804	2462	3358	0.00173	0.00177	5.55E-04
[43,]	Barrancos	10	19	54	36.699	273	139	0.00135	0.00119	4.07E-04
[44,]	Barreiro	29	2149	2217	129.167	5	0	0.00163	0.00170	5.40E-04
[45,]	Batalha	13	475	772	28.283	0	207	0.00157	0.00167	5.47E-04
[46,]	Beja	97	1293	202	1365.639	4004	4828	0.00154	0.00148	4.93E-04
[47,]	Belmonte	7	122	232	16.746	0	0	0.00140	0.00138	4.73E-04
[48,]	Benavente	23	154	1048	109.23	4	0	0.00163	0.00166	5.39E-04
[49,]	Bombarral	10	93	340	12.202	1	304	0.00150	0.00152	5.31E-04
[50,]	Borba	7	33	304	16.683	300	610	0.00133	0.00130	4.63E-04
[51,]	Boticas	5	10	168	4.273	0	0	0.00147	0.00142	5.10E-04
[52,]	Braga	121	7001	2807	2800.065	10223	11248	0.00178	0.00181	5.57E-04
[53,]	Bragança	142	1371	177	1423.777	1789	4335	0.00159	0.00148	4.96E-04
[54,]	Cabeceiras de Basto	12	61	281	42.768	3	3	0.00157	0.00148	5.00E-04
[55,]	Cadaval	11	44	430	35.69	0	0	0.00149	0.00154	5.36E-04
[56,]	Caldas da Rainha	99	1657	969	1120.824	3437	2399	0.00166	0.00170	5.49E-04
[57,]	Calheta (R.A.A.)	9	16	67	51.325	9	306	0.00142	0.00103	2.75E-04
[58,]	Calheta (R.A.M.)	5	16	195	20.73	18	604	0.00151	0.00156	5.56E-04
[59,]	Câmara de Lobos	8	50	2197	77.074	0	1511	0.00160	0.00171	5.75E-04
[60,]	Caminha	10	304	390	42.891	407	1510	0.00158	0.00153	5.32E-04

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[61,]	Campo Maior	4	23	78	66.008	0	0	0.00142	0.00116	4.06E-04
[62,]	Cantanhede	16	698	1029	165.446	1619	5816	0.00166	0.00163	5.45E-04
[63,]	Carrazeda de Ansiães	6	49	60	5.462	0	0	0.00145	0.00111	3.53E-04
[64,]	Carregal do Sal	8	39	210	25.597	18	1	0.00150	0.00152	5.25E-04
[65,]	Cartaxo	14	213	636	26.336	27	16	0.00157	0.00160	5.31E-04
[66,]	Cascais	83	4312	8154	1061.412	4386	16649	0.00172	0.00177	5.45E-04
[67,]	Castanheira de Pêra	3	12	99	3.933	0	0	0.00136	0.00114	4.40E-04
[68,]	Castelo Branco	187	890	307	1912.815	2205	3306	0.00156	0.00156	5.09E-04
[69,]	Castelo de Paiva	14	180	257	54.567	1	297	0.00157	0.00154	5.25E-04
[70,]	Castelo de Vide	7	23	127	11.765	0	0	0.00132	0.00126	4.40E-04
[71,]	Castro Daire	9	28	219	83.292	0	0	0.00158	0.00158	5.39E-04
[72,]	Castro Marim	3	110	355	11.899	209	240	0.00143	0.00145	5.14E-04
[73,]	Castro Verde	6	30	83	17.076	3	27	0.00138	0.00123	4.30E-04
[74,]	Celorico da Beira	7	26	127	8.802	0	23	0.00143	0.00139	4.71E-04
[75,]	Celorico de Basto	13	104	607	96.722	875	947	0.00153	0.00156	5.33E-04
[76,]	Chamusca	11	45	267	35.379	6	1	0.00149	0.00143	4.98E-04
[77,]	Chaves	41	444	393	366.163	1132	1523	0.00164	0.00161	5.33E-04
[78,]	Cinfães	11	36	247	71.734	29	0	0.00157	0.00153	5.13E-04
[79,]	Coimbra	264	10650	765	7440.071	18546	19614	0.00177	0.00172	5.42E-04
[80,]	Condeixa-a-Nova	12	98	1306	90.604	272	0	0.00155	0.00165	5.40E-04
[81,]	Constância	7	102	192	15.038	10	21	0.00140	0.00144	5.08E-04
[82,]	Coruche	12	118	272	59.365	9	303	0.00151	0.00151	5.03E-04
[83,]	Corvo	2	2	2	4.81	0	0	0.00111	0.00066	8.35E-05
[84,]	Covilhã	207	1556	359	1499.474	1573	2052	0.00161	0.00147	4.84E-04
[85,]	Crato	8	27	80	18.428	0	0	0.00130	0.00122	4.28E-04
[86,]	Cuba	12	76	214	24.925	0	276	0.00134	0.00134	4.78E-04
[87,]	Elvas	51	120	196	336.2	656	600	0.00145	0.00144	4.77E-04
[88,]	Entroncamento	25	708	732	213.242	609	864	0.00163	0.00161	5.25E-04
[89,]	Espinho	38	2051	1110	188.737	0	0	0.00156	0.00170	5.49E-04
[90,]	Esposende	18	362	1031	48.114	0	0	0.00164	0.00167	5.45E-04
[91,]	Estarreja	15	356	793	31.194	340	610	0.00159	0.00160	5.45E-04
[92,]	Estremoz	16	307	168	128.211	972	1940	0.00148	0.00147	4.89E-04

Node	Municipality	Node	In -	Out -	Betweenness			Closeness		
[93,]	Évora	171	1895	271	3162.877	5417	8337	0.00162	0.00157	5.22E-04
[94,]	Fafe	27	410	945	102.544	711	1267	0.00162	0.00167	5.48E-04
[95,]	Faro	144	3604	496	1794.009	3759	7856	0.00156	0.00157	5.35E-04
[96,]	Felgueiras	42	1320	1175	337.465	139	355	0.00173	0.00168	5.49E-04
[97,]	Ferreira do Alentejo	13	24	258	69.155	0	0	0.00133	0.00135	4.83E-04
[98,]	Ferreira do Zêzere	8	71	233	32.4	11	299	0.00151	0.00147	5.14E-04
[99,]	Figueira da Foz	34	286	914	296.873	24	0	0.00169	0.00171	5.47E-04
[100,]	Figueira de Castelo Rodrigo	5	16	89	10.242	1	1	0.00136	0.00124	4.11E-04
[101,]	Figueiró dos Vinhos	9	130	129	85.993	318	1190	0.00141	0.00132	4.76E-04
[102,]	Fornos de Algodres	10	69	63	27.065	5	3	0.00141	0.00122	3.67E-04
[103,]	Freixo de Espada à Cinta	3	3	57	15.734	3	0	0.00137	0.00117	3.54E-04
[104,]	Fronteira	7	15	97	12.582	20	29	0.00128	0.00118	4.10E-04
[105,]	Funchal	27	6416	499	846.225	4290	6039	0.00173	0.00176	5.74E-04
[106,]	Fundão	16	193	487	129.766	417	609	0.00155	0.00143	4.87E-04
[107,]	Gavião	9	41	102	34.64	386	2451	0.00137	0.00123	4.12E-04
[108,]	Góis	8	19	132	32.684	12	0	0.00130	0.00130	4.71E-04
[109,]	Golegã	7	61	217	10.22	2	305	0.00147	0.00141	4.98E-04
[110,]	Gondomar	27	1258	9005	239.664	666	907	0.00177	0.00177	5.56E-04
[111,]	Gouveia	17	125	208	54.504	32	934	0.00150	0.00138	4.74E-04
[112,]	Grândola	11	111	153	24.892	859	1536	0.00140	0.00137	4.57E-04
[113,]	Guarda	139	832	403	1923.147	2959	4837	0.00167	0.00161	4.97E-04
[114,]	Guimarães	88	2815	4477	1298.921	2400	5525	0.00176	0.00180	5.58E-04
[115,]	Horta	6	6	42	127.268	5	2	0.00152	0.00119	3.45E-04
[116,]	Idanha-a-Nova	47	137	74	108.183	0	0	0.00139	0.00129	4.55E-04
[117,]	Ílhavo	14	270	1721	48.321	0	100	0.00166	0.00164	5.51E-04
[118,]	Lagoa	22	589	1021	156.065	477	558	0.00154	0.00147	5.18E-04
[119,]	Lagoa (R.A.A.)	0	0	685	0	0	0	0.00000	0.00000	0
[120,]	Lagos	14	360	365	60.753	907	1217	0.00156	0.00148	5.03E-04
[121,]	Lajes das Flores	0	0	75	0	0	0	0.00000	0.00000	0
[122,]	Lajes do Pico	6	11	40	34.175	2	0	0.00139	0.00097	2.58E-04
[123,]	Lamego	65	532	379	917.665	1357	1689	0.00164	0.00161	5.19E-04
[124,]	Leiria	178	2866	2181	3592.343	4535	7574	0.00179	0.00182	5.56E-04

Node	Municipality	Node	In -	Out -	Betweenness			Closeness		
[125,]	Lisboa	288	65323	3904	10555.887	47081.5	53262	0.00183	0.00176	5.42E-04
[126,]	Loulé	33	653	1206	446.819	1358	3392	0.00167	0.00157	5.33E-04
[127,]	Loures	48	3264	10381	981.361	11275	12101	0.00184	0.00176	5.43E-04
[128,]	Lourinhã	17	219	735	179.44	296	610	0.00166	0.00158	5.33E-04
[129,]	Lousã	18	129	632	292.235	275	260	0.00152	0.00162	5.40E-04
[130,]	Lousada	31	589	1615	138.381	291	13	0.00164	0.00169	5.51E-04
[131,]	Mação	10	74	144	289.257	590	923	0.00152	0.00127	4.40E-04
[132,]	Macedo de Cavaleiros	37	156	324	206.442	11	1407	0.00152	0.00137	4.82E-04
[133,]	Machico	7	175	630	60.068	34	306	0.00160	0.00165	5.67E-04
[134,]	Madalena	12	76	35	30.098	793	1162	0.00142	0.00113	3.06E-04
[135,]	Mafra	23	550	3206	100.562	1446	1801	0.00163	0.00173	5.42E-04
[136,]	Maia	72	4928	8161	975.409	1527	9127	0.00174	0.00176	5.56E-04
[137,]	Mangualde	11	132	351	48.541	367	1792	0.00152	0.00165	5.58E-04
[138,]	Manteigas	10	22	55	8.399	0	0	0.00137	0.00120	3.76E-04
[139,]	Marco de Canaveses	18	182	1096	94.654	661	889	0.00163	0.00169	5.51E-04
[140,]	Marinha Grande	15	678	695	21.73	0	0	0.00151	0.00170	5.50E-04
[141,]	Marvão	8	20	124	8.364	0	0	0.00135	0.00126	4.39E-04
[142,]	Matosinhos	62	4191	8214	1300.518	1453	9405	0.00177	0.00176	5.55E-04
[143,]	Mealhada	19	273	1108	101.541	339	305	0.00156	0.00164	5.49E-04
[144,]	Meda	4	17	54	32.183	13	307	0.00135	0.00109	3.56E-04
[145,]	Melgaço	17	69	107	35.24	2	0	0.00150	0.00131	4.29E-04
[146,]	Mértola	8	20	93	29.41	19	6	0.00132	0.00124	4.45E-04
[147,]	Mesão Frio	6	192	120	9.934	11	7	0.00135	0.00125	4.68E-04
[148,]	Mira	12	91	458	59.046	6	0	0.00154	0.00154	5.34E-04
[149,]	Miranda do Corvo	8	88	712	46.215	2	555	0.00149	0.00162	5.40E-04
[150,]	Miranda do Douro	6	50	63	23.401	3	1	0.00147	0.00124	4.24E-04
[151,]	Mirandela	80	520	244	495.708	988	2131	0.00159	0.00147	5.01E-04
[152,]	Mogadouro	9	17	130	221.865	56	306	0.00142	0.00132	4.57E-04
[153,]	Moimenta da Beira	10	269	195	42.114	345	611	0.00151	0.00148	4.99E-04
[154,]	Moita	18	697	3131	247.679	507	211	0.00166	0.00170	5.39E-04
[155,]	Monção	14	94	180	111.963	4	5	0.00159	0.00145	4.86E-04
[156,]	Monchique	3	19	165	0.862	0	0	0.00130	0.00137	5.00E-04

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[157,]	Mondim de Basto	5	82	160	6.882	241	308	0.00145	0.00134	4.60E-04
[158,]	Monforte	3	10	112	40.605	0	0	0.00135	0.00124	4.38E-04
[159,]	Montalegre	9	36	160	44.593	4	4	0.00151	0.00148	5.05E-04
[160,]	Montemor-o-Novo	13	41	359	58.239	13	0	0.00151	0.00145	5.11E-04
[161,]	Montemor-o-Velho	16	226	1228	65.877	507	4986	0.00155	0.00166	5.45E-04
[162,]	Montijo	26	660	1721	168.163	344	887	0.00158	0.00169	5.39E-04
[163,]	Mora	17	37	53	41.668	0	0	0.00130	0.00121	4.25E-04
[164,]	Mortágua	13	58	132	57.688	3	4	0.00151	0.00142	4.82E-04
[165,]	Moura	10	88	125	71.655	22	7	0.00144	0.00134	4.44E-04
[166,]	Mourão	3	4	83	2.364	0	0	0.00131	0.00122	4.40E-04
[167,]	Murça	7	58	71	15.345	0	0	0.00144	0.00127	4.47E-04
[168,]	Murtosa	10	107	369	11.025	0	0	0.00151	0.00146	5.30E-04
[169,]	Nazaré	15	117	515	40.088	0	0	0.00153	0.00159	5.35E-04
[170,]	Nelas	13	111	290	54.825	6	8	0.00156	0.00161	5.50E-04
[171,]	Nisa	19	76	144	148.195	26	586	0.00145	0.00126	4.40E-04
[172,]	Nordeste	4	8	41	15.735	0	0	0.00129	0.00130	4.69E-04
[173,]	Óbidos	12	213	561	109.308	181	0	0.00150	0.00158	5.42E-04
[174,]	Odemira	15	110	155	145.062	21	112	0.00150	0.00145	4.88E-04
[175,]	Odivelas	44	2516	6631	538.353	1	0	0.00173	0.00173	5.42E-04
[176,]	Oeiras	87	6618	10199	1601.077	520	28305	0.00174	0.00175	5.44E-04
[177,]	Oleiros	11	14	79	41.063	0	0	0.00142	0.00126	4.10E-04
[178,]	Olhão	11	242	1326	64.749	1000	1513	0.00154	0.00152	5.33E-04
[179,]	Oliveira de Azeméis	34	747	2248	390.332	377	561	0.00171	0.00169	5.48E-04
[180,]	Oliveira de Frades	9	155	250	23.981	20	33	0.00149	0.00138	5.06E-04
[181,]	Oliveira do Bairro	13	562	834	16.597	286	5236	0.00152	0.00162	5.45E-04
[182,]	Oliveira do Hospital	50	251	190	351.689	174	379	0.00155	0.00150	5.10E-04
[183,]	Ourém	25	1098	792	170.211	1283	5229	0.00169	0.00170	5.47E-04
[184,]	Ourique	9	26	65	139.648	258	1	0.00139	0.00112	3.78E-04
[185,]	Ovar	26	558	2016	374.954	559	658	0.00171	0.00173	5.55E-04
[186,]	Paços de Ferreira	16	479	1441	72.829	413	3898	0.00163	0.00169	5.53E-04
[187,]	Palmela	26	1420	2682	148.587	1689	1999	0.00160	0.00170	5.40E-04
[188,]	Pampilhosa da Serra	8	16	54	48.105	0	0	0.00135	0.00123	4.05E-04

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[189,]	Paredes	71	1684	3021	732.713	4318	18852	0.00169	0.00173	5.57E-04
[190,]	Paredes de Coura	11	50	91	12.616	0	0	0.00141	0.00123	3.83E-04
[191,]	Pedrógão Grande	14	118	91	178.697	360	338	0.00135	0.00120	4.15E-04
[192,]	Penacova	15	76	716	57.185	432	839	0.00141	0.00163	5.43E-04
[193,]	Penafiel	26	910	2117	114.467	1949	6705	0.00170	0.00173	5.56E-04
[194,]	Penalva do Castelo	5	37	179	123.183	307	304	0.00144	0.00154	5.29E-04
[195,]	Penamacor	6	12	57	4.668	0	0	0.00141	0.00121	4.13E-04
[196,]	Penedono	4	34	93	9.857	4	9	0.00147	0.00137	4.54E-04
[197,]	Penela	14	87	286	104.185	2	0	0.00144	0.00156	5.29E-04
[198,]	Peniche	77	406	365	408.427	6	16	0.00148	0.00154	5.22E-04
[199,]	Peso da Régua	35	535	281	393.977	954	1938	0.00154	0.00147	5.13E-04
[200,]	Pinhel	6	16	221	45.783	0	0	0.00151	0.00142	4.76E-04
[201,]	Pombal	31	697	834	429.228	1278	5353	0.00171	0.00171	5.52E-04
[202,]	Ponta Delgada	35	1998	308	1203.201	3731.5	4622	0.00163	0.00164	5.50E-04
[203,]	Ponta do Sol	5	30	272	9.097	0	908	0.00139	0.00158	5.60E-04
[204,]	Ponte da Barca	9	125	226	13.126	0	0	0.00148	0.00139	4.66E-04
[205,]	Ponte de Lima	40	376	872	187.982	313	607	0.00159	0.00160	5.42E-04
[206,]	Ponte de Sor	6	116	159	37.246	333	1587	0.00146	0.00141	4.49E-04
[207,]	Portalegre	130	974	140	1186.881	3297	3329	0.00151	0.00141	4.59E-04
[208,]	Portel	32	76	216	397.558	246	72	0.00137	0.00141	5.06E-04
[209,]	Portimão	47	1283	538	549.397	1812	1996	0.00155	0.00154	5.21E-04
[210,]	Porto	219	42416	2315	6132.188	29983	33688	0.00181	0.00178	5.55E-04
[211,]	Porto de Mós	24	438	834	75.839	8	52	0.00158	0.00166	5.45E-04
[212,]	Porto Moniz	11	20	41	54.189	0	0	0.00132	0.00133	4.61E-04
[213,]	Porto Santo	4	7	23	3.306	0	0	0.00138	0.00122	3.97E-04
[214,]	Póvoa de Lanhoso	46	717	582	271.851	681	635	0.00157	0.00166	5.47E-04
[215,]	Póvoa de Varzim	22	1432	1959	174.716	188	56	0.00165	0.00172	5.53E-04
[216,]	Povoação	4	7	43	13.718	0	0	0.00140	0.00129	4.63E-04
[217,]	Proença-a-Nova	2	44	100	2.818	2	327	0.00152	0.00125	4.26E-04
[218,]	Redondo	6	16	171	7.8	0	0	0.00129	0.00139	5.01E-04
[219,]	Reguengos de Monsaraz	8	152	131	61.875	311	900	0.00141	0.00137	4.94E-04
[220,]	Resende	14	76	119	43.904	0	0	0.00154	0.00138	4.51E-04

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[221,]	Ribeira Brava	7	257	414	26.606	297	1210	0.00149	0.00163	5.67E-04
[222,]	Ribeira de Pena	8	39	73	42.571	10	4	0.00142	0.00123	4.13E-04
[223,]	Ribeira Grande	10	90	1135	543.293	1043	1222	0.00153	0.00157	5.47E-04
[224,]	Rio Maior	75	494	418	345.645	73	122	0.00153	0.00154	5.28E-04
[225,]	Sabrosa	12	68	180	40.23	328	642	0.00151	0.00142	5.11E-04
[226,]	Sabugal	13	78	165	40.629	0	0	0.00146	0.00137	4.60E-04
[227,]	Salvaterra de Magos	12	317	433	23.339	262	914	0.00156	0.00157	5.24E-04
[228,]	Santa Comba Dão	7	74	189	31.351	8	33	0.00151	0.00147	5.09E-04
[229,]	Santa Cruz	20	203	3383	450.21	1933	1306	0.00159	0.00172	5.74E-04
[230,]	Santa Cruz da Graciosa	1	1	13	1.637	0	0	0.00134	0.00098	2.36E-04
[231,]	Santa Cruz das Flores	3	67	4	2.394	0	0	0.00127	0.00073	8.54E-05
[232,]	Santa Maria da Feira	43	1594	5930	939.293	4614	13650	0.00184	0.00182	5.61E-04
[233,]	Santa Marta de Penaguião	3	35	378	13.619	3	0	0.00139	0.00146	5.19E-04
[234,]	Santana	21	47	183	80.692	14	0	0.00145	0.00152	5.40E-04
[235,]	Santarém	111	2079	821	1422.591	3621	9026	0.00166	0.00168	5.41E-04
[236,]	Santiago do Cacém	13	143	392	75.644	278	1267	0.00149	0.00152	5.19E-04
[237,]	Santo Tirso	25	2148	2006	274.245	495	3827	0.00174	0.00174	5.56E-04
[238,]	São Brás de Alportel	6	85	308	8.501	0	0	0.00137	0.00144	5.22E-04
[239,]	São João da Madeira	29	2886	554	105.404	0	47	0.00160	0.00163	5.37E-04
[240,]	São João da Pesqueira	16	67	135	263.91	317	313	0.00152	0.00138	4.40E-04
[241,]	São Pedro do Sul	19	163	266	39.46	242	641	0.00148	0.00157	5.40E-04
[242,]	São Roque do Pico	10	26	54	58.035	3	9	0.00137	0.00101	2.87E-04
[243,]	São Vicente	9	39	102	177.484	301	2646	0.00142	0.00148	5.31E-04
[244,]	Sardoal	3	113	113	4.504	0	0	0.00144	0.00138	4.91E-04
[245,]	Sátão	7	59	258	22.249	814	1266	0.00152	0.00161	5.50E-04
[246,]	Seia	69	275	284	570.696	149	987	0.00159	0.00148	4.95E-04
[247,]	Seixal	28	1671	6679	262.356	1999	7530	0.00176	0.00173	5.42E-04
[248,]	Sernancelhe	12	58	231	102.266	24	375	0.00151	0.00145	4.92E-04
[249,]	Serpa	20	57	223	117.965	121	163	0.00145	0.00133	4.75E-04
[250,]	Sertã	14	148	144	227.814	213	347	0.00150	0.00136	4.45E-04
[251,]	Sesimbra	13	415	2116	44.663	310	0	0.00160	0.00169	5.39E-04
[252,]	Setúbal	104	3297	2664	1530.649	4245	7317	0.00171	0.00172	5.41E-04

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[253,]	Sever do Vouga	7	34	241	10.823	1	0	0.00149	0.00146	5.06E-04
[254,]	Silves	30	468	1035	330.173	210	2085	0.00161	0.00146	5.21E-04
[255,]	Sines	16	194	167	67.071	0	7	0.00148	0.00143	4.89E-04
[256,]	Sintra	46	2482	18147	1681.031	6561	11480	0.00195	0.00178	5.45E-04
[257,]	Sobral de Monte Agraço	10	220	611	7.193	3	0	0.00149	0.00157	5.23E-04
[258,]	Soure	12	224	684	36.279	747	4437	0.00148	0.00165	5.44E-04
[259,]	Sousel	6	17	166	22.008	12	5	0.00132	0.00131	4.71E-04
[260,]	Tábua	12	53	297	33.279	222	18	0.00147	0.00143	4.93E-04
[261,]	Tabuaço	6	8	103	14.076	0	0	0.00142	0.00127	4.64E-04
[262,]	Tarouca	8	30	234	24.332	1	0	0.00149	0.00145	5.02E-04
[263,]	Tavira	16	187	532	94.384	129	1221	0.00149	0.00149	5.26E-04
[264,]	Terras de Bouro	7	50	166	11.715	0	0	0.00147	0.00149	5.09E-04
[265,]	Tomar	119	1220	479	1445.786	2567	6021	0.00161	0.00163	5.31E-04
[266,]	Tondela	25	189	469	188.083	202	609	0.00164	0.00168	5.61E-04
[267,]	Torre de Moncorvo	10	25	105	112.182	7	2	0.00150	0.00131	4.20E-04
[268,]	Torres Novas	73	711	871	716.5	418	3417	0.00159	0.00163	5.30E-04
[269,]	Torres Vedras	27	1450	1403	137.494	1255	1818	0.00169	0.00169	5.39E-04
[270,]	Trancoso	18	190	128	246.529	379	923	0.00149	0.00133	4.33E-04
[271,]	Trofa	21	756	1802	88.838	6	4652	0.00162	0.00171	5.54E-04
[272,]	Vagos	21	503	739	189.541	188	208	0.00158	0.00158	5.45E-04
[273,]	Vale de Cambra	13	211	378	34.966	1	0	0.00155	0.00155	5.15E-04
[274,]	Valença	23	181	345	40.043	291	613	0.00152	0.00137	4.61E-04
[275,]	Valongo	39	2214	4378	249.989	344	4592	0.00165	0.00174	5.56E-04
[276,]	Valpaços	6	13	304	23.553	0	0	0.00152	0.00144	5.17E-04
[277,]	Velas	7	55	27	62.669	176	296	0.00139	0.00111	2.88E-04
[278,]	Vendas Novas	12	174	162	19.996	7	277	0.00139	0.00142	4.73E-04
[279,]	Viana do Alentejo	13	79	170	54.062	12	293	0.00133	0.00140	5.03E-04
[280,]	Viana do Castelo	65	1822	1110	676.151	2391	4397	0.00169	0.00170	5.48E-04
[281,]	Vidigueira	9	44	232	65.668	2	6	0.00146	0.00134	4.78E-04
[282,]	Vieira do Minho	11	28	322	18.042	302	306	0.00146	0.00159	5.38E-04
[283,]	Vila da Praia da Vitória	3	210	426	25.263	33	42	0.00154	0.00144	5.04E-04
[284,]	Vila de Rei	11	27	76	50.936	0	0	0.00137	0.00117	4.05E-04

NODE	MUNICIPALITY	NODE	IN -	OUT -	BETWEENNESS			CLOSENESS		
[285,]	Vila do Bispo	6	36	216	8.058	0	0	0.00134	0.00135	4.89E-04
[286,]	Vila do Conde	50	1287	3407	350.008	434	286	0.00171	0.00173	5.55E-04
[287,]	Vila do Porto	7	8	21	30.753	0	0	0.00142	0.00113	3.07E-04
[288,]	Vila Flor	7	20	115	31.421	129	608	0.00148	0.00122	4.15E-04
[289,]	Vila Franca de Xira	34	1253	5512	309.685	4406	8523	0.00169	0.00175	5.44E-04
[290,]	Vila Franca do Campo	4	19	239	5.563	0	0	0.00141	0.00149	5.38E-04
[291,]	Vila Nova da Barquinha	8	113	399	44.978	22	237	0.00153	0.00146	5.11E-04
[292,]	Vila Nova de Cerveira	16	292	190	66.319	3	911	0.00149	0.00136	4.65E-04
[293,]	Vila Nova de Famalicão	62	3381	4259	441.299	457	8849	0.00172	0.00179	5.58E-04
[294,]	Vila Nova de Foz Côa	4	6	100	29.652	2	1	0.00152	0.00133	3.90E-04
[295,]	Vila Nova de Gaia	97	3296	10783	2230.336	8246	15813	0.00186	0.00183	5.58E-04
[296,]	Vila Nova de Paiva	11	98	130	94.013	289	566	0.00152	0.00151	5.21E-04
[297,]	Vila Nova de Poiares	14	74	236	36.867	0	3	0.00143	0.00154	5.29E-04
[298,]	Vila Pouca de Aguiar	11	30	271	23.81	0	0	0.00149	0.00143	5.14E-04
[299,]	Vila Real	160	2089	391	2093.629	4630	6660	0.00164	0.00161	5.33E-04
[300,]	Vila Real de Santo António	13	309	325	42.474	601	750	0.00146	0.00142	5.08E-04
[301,]	Vila Velha de Ródão	3	9	90	9.26	64	0	0.00137	0.00135	4.81E-04
[302,]	Vila Verde	17	356	1800	148.933	898	610	0.00162	0.00173	5.54E-04
[303,]	Vila Viçosa	8	251	88	23.629	30	316	0.00139	0.00124	4.25E-04
[304,]	Vimioso	2	2	100	4.644	1	0	0.00138	0.00127	4.69E-04
[305,]	Vinhais	4	7	154	8.274	0	0	0.00145	0.00129	4.68E-04
[306,]	Viseu	157	2162	703	3460.805	9888	11569	0.00174	0.00183	5.70E-04
[307,]	Vizela	13	370	704	46.925	5	0	0.00159	0.00165	5.47E-04
[308,]	Vouzela	15	185	291	47.966	245	635	0.00151	0.00157	5.38E-04

TABLE 23 – Outcomes for each Portuguese municipality, commuting interactions due to academic reasons variable, 2011: Node Degree, In-Degree, Out-Degree, and Betweenness and Closeness, considering different levels of alpha

1.2. European Union Domains Outcomes

EU Rank	World Rank	Country	HDI	EU Rank	World Rank	Country	HDI
[1,]	[4,]	Denmark	0.92	**[15,]**	[28,]	Czech Republic	0.866
[2,]	[5,]	Netherlands	0.919	**[16,]**	[29,]	Greece	0.864
[3,]	[6,]	Germany	0.911	**[17,]**	[32,]	Cyprus	0.852
[4,]	[6,]	Ireland	0.909	**[18,]**	[30,]	Estonia	0.849
[5,]	[14,]	Sweden	0.903	**[19,]**	[36,]	Poland	0.833
[6,]	[14,]	United Kingdom	0.901	**[20,]**	[35,]	Slovakia	0.832
[7,]	[19,]	Luxembourg	0.888	**[21,]**	[37,]	Lithuania	0.831
[8,]	[21,]	Belgium	0.886	**[22,]**	[43,]	Portugal	0.825
[9,]	[22,]	France	0.884	**[23,]**	[44,]	Hungary	0.823
[10,]	[23,]	Austria	0.881	**[24,]**	[37,]	Malta	0.822
[11,]	[24,]	Finland	0.881	**[25,]**	[47,]	Croatia	0.814
[12,]	[25,]	Slovenia	0.877	**[26,]**	[46,]	Latvia	0.812
[13,]	[27,]	Italy	0.873	**[27,]**	[52,]	Romania	0.786
[14,]	[26,]	Spain	0.87	**[28,]**	[59,]	Bulgaria	0.775

Table 24 – Human Development Index Rank, by EU Countries, 2011

Source: United Nations Development Programme, Human Development Reports, Available at: hdr.undp.org/en/data

#	Country	Node Degree	Import Value	EU HDI Rank	Betweenness			Closeness		
					alpha=0	alpha=0.5	alpha=1	alpha=0	alpha=0.5	alpha=1
[1,]	Austria	27	117,127,785	[10,]	15.103	0	0	0.02941	0.02931	0.00653
[2,]	Belgium	27	287,386,056	[8,]	6.071	0	2	0.02857	0.03026	0.00655
[3,]	Bulgaria	15	17,057,981	[28,]	6.145	0	0	0.02857	0.01974	0.00513
[4,]	Cyprus	6	4,769,761	[17,]	0.964	0	0	0.02857	0.01303	0.00368
[5,]	Czech Republic	27	91,421,752	[15,]	4.996	0	0	0.02857	0.02805	0.00646
[6,]	Germany	27	589,267,427	[3,]	4.996	549	634	0.02857	0.03481	0.00663
[7,]	Denmark	27	61,484,485	[1,]	17.207	0	0	0.02857	0.02664	0.00633
[8,]	Spain	27	174,275,440	[14,]	5.270	10	52	0.02857	0.02931	0.0065
[9,]	Estonia	5	8,403,425	[18,]	0.389	0	0	0.02857	0.01665	0.00411
[10,]	Finland	26	45,133,522	[11,]	16.306	0	0	0.02857	0.02507	0.00614
[11,]	France	27	387,417,987	[9,]	4.996	2	4	0.02857	0.03191	0.0066
[12,]	United Kingdom	27	304,351,710	[6,]	6.109	26	26	0.02857	0.03104	0.00656
[13,]	Greece	15	31,247,312	[16,]	5.916	27	52	0.02857	0.0237	0.00593
[14,]	Croatia	5	12,274,744	[25,]	26.318	26	26	0.02941	0.01968	0.0051
[15,]	Hungary	27	60,794,875	[23,]	15.103	0	0	0.02941	0.02634	0.00635
[16,]	Ireland	26	35,517,562	[4,]	5.715	0	0	0.02857	0.02368	0.00624
[17,]	Italy	27	281,267,202	[13,]	16.117	121	147	0.02941	0.03213	0.00659
[18,]	Lithuania	11	16,439,774	[21,]	7.705	0	0	0.02857	0.02015	0.0053
[19,]	Luxembourg	8	16,568,683	[7,]	0.699	0	0	0.02857	0.02174	0.00572
[20,]	Latvia	5	9,337,390	[26,]	1.654	0	0	0.02857	0.01722	0.00438
[21,]	Malta	1	3,018,772	[24,]	0.472	0	0	0.02941	0.01533	0.00554
[22,]	Netherlands	27	226,642,706	[2,]	6.109	0	0	0.02857	0.03007	0.00655
[23,]	Poland	27	116,038,025	[19,]	4.506	18	23	0.02857	0.02881	0.0065
[24,]	Portugal	20	50,979,148	[22,]	2.999	0	0	0.02857	0.0239	0.00622
[25,]	Romania	23	50,623,947	[27,]	2.574	0	0	0.02857	0.02499	0.00615
[26,]	Slovakia	26	38,032,804	[20,]	3.449	0	0	0.02857	0.02455	0.00615
[27,]	Slovenia	17	19,106,981	[12,]	13.458	0	0	0.02941	0.02148	0.00553
[28,]	Sweden	27	105,037,566	[5,]	16.652	49	49	0.02857	0.02943	0.00649

Table 25 – Outcomes for each EU-28 country, imports trade variable, 2011: Node Degree, Import Value (Out-Degree), HDI EU Rank and Betweenness and Closeness, considering different levels of alpha

#	Country	Node Degree	Export Value	EU HDI Rank	Betweenness			Closeness		
					alpha=0	alpha=0.5	alpha=1	alpha=0	alpha=0.5	alpha=1
[1,]	Austria	27	103,929,210	[10,]	6.392	0	0	0.02941	0.03907	0.02269
[2,]	Belgium	27	307,170,843	[8,]	8.226	23	24	0.02941	0.04226	0.02342
[3,]	Bulgaria	15	16,247,909	[28,]	4.807	0	0	0.02941	0.02376	0.01137
[4,]	Cyprus	7	489,970	[17,]	3.515	0	0	0.02941	0.00601	0.00068
[5,]	Czech Republic	27	127,038,946	[15,]	8.185	0	0	0.02941	0.03955	0.02285
[6,]	Germany	27	768,023,785	[3,]	7.604	571	637	0.02941	0.05148	0.02450
[7,]	Denmark	27	61,104,182	[1,]	8.626	0	0	0.02941	0.03372	0.01981
[8,]	Spain	27	185,047,654	[14,]	4.617	4	52	0.02941	0.03893	0.02219
[9,]	Estonia	5	8,608,412	[18,]	1.267	0	0	0.02941	0.01753	0.00781
[10,]	Finland	24	33,207,197	[11,]	11.728	0	23	0.02941	0.02820	0.01554
[11,]	France	27	330,093,117	[9,]	7.604	1	50	0.02941	0.04353	0.02360
[12,]	United Kingdom	27	216,522,427	[6,]	26.204	53	52	0.02941	0.04122	0.02291
[13,]	Greece	23	14,238,944	[16,]	7.837	25	52	0.02941	0.02272	0.01046
[14,]	Croatia	8	7,354,279	[25,]	5.086	0	0	0.02941	0.01881	0.00830
[15,]	Hungary	26	77,892,668	[23,]	6.025	0	0	0.02941	0.03642	0.02142
[16,]	Ireland	15	68,419,019	[4,]	3.366	0	0	0.02941	0.03057	0.01969
[17,]	Italy	27	271,395,658	[13,]	6.392	68	119	0.02941	0.04362	0.02337
[18,]	Lithuania	11	11,809,224	[21,]	3.995	0	25	0.02941	0.02204	0.00953
[19,]	Luxembourg	6	12,667,673	[7,]	0.473	0	0	0.02941	0.02494	0.01312
[20,]	Latvia	7	7,658,007	[26,]	2.425	0	0	0.02941	0.01646	0.00558
[21,]	Malta	3	1,157,001	[24,]	0.592	0	0	0.02941	0.00988	0.00168
[22,]	Netherlands	27	325,305,195	[2,]	8.226	0	0	0.02941	0.04300	0.02367
[23,]	Poland	27	137,088,024	[19,]	14.520	48	49	0.02941	0.04058	0.02292
[24,]	Portugal	22	42,260,719	[22,]	5.435	0	0	0.02941	0.02941	0.01785
[25,]	Romania	26	41,617,264	[27,]	5.931	0	0	0.02941	0.03171	0.01846
[26,]	Slovakia	25	61,180,405	[20,]	5.689	0	0	0.02941	0.03321	0.01974
[27,]	Slovenia	13	21,164,217	[12,]	5.541	0	0	0.02941	0.02754	0.01508
[28,]	Sweden	27	87,350,922	[5,]	15.692	45	24	0.02941	0.03492	0.02025

Table 26 – Outcomes for each EU-28 country, exports trade variable, 2011: Node Degree, Import Value (Out-Degree), HDI EU Rank and Betweenness and Closeness, considering different levels of alpha

#	Country	Node Degree	In - Degree	Out- Degree	EU HDI Rank	Betweenness alpha=0	Betweenness alpha=0.5	Betweenness alpha=1	Closeness alpha=0	Closeness alpha=0.5	Closeness alpha=1
[1,]	Austria	11	14,351	11,337	[10,]	23.036	41	59	0.03571	0.03897	0.01156
[2,]	Belgium	12	54,197	92,667	[8,]	10.683	57	148	0.03333	0.04608	0.01266
[3,]	Bulgaria	8	1,212	15,801	[28,]	0.000	0	0	0.03448	0.01177	0.00149
[4,]	Cyprus	12	2,219	839	[17,]	0.000	0	0	0.04000	0.04910	0.01256
[5,]	Czech Republic	8	949	0	[15,]	7.077	1	0	0.04348	0.04343	0.01407
[6,]	Germany	12	4,908	18,544	[3,]	31.363	203	245	0.03125	0.03970	0.01180
[7,]	Denmark	13	3,652	0	[1,]	18.017	19	19	0.03125	0.01352	0.00201
[8,]	Spain	14	101,531	97,561	[14,]	5.808	0	0	0.03571	0.03416	0.01134
[9,]	Estonia	9	4,306	339	[18,]	1.888	0	0	0.02857	0.04449	0.01277
[10,]	Finland	14	39,989	68,970	[11,]	3.485	0	0	0.03030	0.04256	0.01238
[11,]	France	9	899	11,383	[9,]	4.792	4	4	0.03448	0.02803	0.01042
[12,]	United Kingdom	5	126	0	[6,]	13.800	62	70	0.03704	0.02147	0.00572
[13,]	Greece	4	129	143	[16,]	3.848	0	0	0.02857	0.04187	0.01238
[14,]	Croatia	10	2,350	0	[25,]	0.000	0	0	0.04348	0.04610	0.01274
[15,]	Hungary	5	3,108	8,112	[23,]	13.544	0	0	0.03030	0.04876	0.01274
[16,]	Ireland	9	6,797	43,285	[4,]	5.353	0	0	0.03333	0.04041	0.01215
[17,]	Italy	14	116,225	30,919	[13,]	41.206	78	79	0.03704	0.03597	0.01147
[18,]	Lithuania	13	96,298	22,354	[21,]	0.000	0	0	0.03846	0.03672	0.01252
[19,]	Luxembourg	6	150	1,762	[7,]	5.747	224	339	0.03030	0.01323	0.00193
[20,]	Latvia	16	14,913	7,939	[26,]	0.000	0	0	0.02941	0.00831	0.00066
[21,]	Malta	8	927	201	[24,]	0.000	0	0	0.03704	0.04164	0.01228
[22,]	Netherlands	6	1,634	0	[2,]	18.165	106	120	0.03571	0.03897	0.01156
[23,]	Poland	10	4,340	1,092	[19,]	38.588	10	18	0.03333	0.04608	0.01266
[24,]	Portugal	12	9,052	24,751	[22,]	12.758	0	0	0.03448	0.01177	0.00149
[25,]	Romania	11	13,786	27,574	[27,]	0.000	0	0	0.04000	0.04910	0.01256
[26,]	Slovakia	14	20,093	35,944	[20,]	5.988	0	0	0.04348	0.04343	0.01407
[27,]	Slovenia	7	1,822	0	[12,]	4.010	0	0	0.03125	0.03970	0.01180
[28,]	Sweden	8	1,554	0	[5,]	20.844	102	108	0.03125	0.01352	0.00201

Table 27 – Outcomes for each EU-28 country, FDI variable, 2011: Node Degree, In-Degree, Out-Degree, HDI EU Rank and Betweenness and Closeness, considering different levels of alpha

#	Country	Node Degree	In - Degree	Out - Degree	EU HDI Rank	Betweenness alpha=0	Betweenness alpha=0.5	Betweenness alpha=1	Closeness alpha=0	Closeness alpha=0.5	Closeness alpha=1
[1,]	Austria	21	2,008	2,137	[10,]	10.463	0	0	0.03571	0.04550	0.02614
[2,]	Belgium	20	8,216	3,935	[8,]	5.356	8	38	0.03333	0.04449	0.02622
[3,]	Bulgaria	20	592	0	[28,]	0.000	0	0	0.00000	0.00000	0.00000
[4,]	Cyprus	20	1,264	1,220	[17,]	3.367	16	49	0.02857	0.02451	0.01072
[5,]	Czech Republic	25	6,435	13,199	[15,]	39.218	421	456	0.03030	0.02432	0.01062
[6,]	Germany	23	11,478	10,809	[3,]	29.006	128	197	0.03704	0.06109	0.02886
[7,]	Denmark	20	669	495	[1,]	7.863	0	0	0.03571	0.03260	0.01733
[8,]	Spain	16	1,513	481	[14,]	0.118	1	2	0.03571	0.05424	0.02786
[9,]	Estonia	19	6,168	8,594	[18,]	4.627	80	214	0.01852	0.00706	0.00105
[10,]	Finland	16	145	18	[11,]	0.267	0	0	0.03226	0.02256	0.01219
[11,]	France	20	714	248	[9,]	22.643	24	23	0.03704	0.05102	0.02731
[12,]	United Kingdom	20	417	1,639	[6,]	10.711	50	72	0.03846	0.05858	0.02858
[13,]	Greece	22	2,352	7,398	[16,]	45.732	152	167	0.03226	0.02548	0.01270
[14,]	Croatia	4	78	130	[25,]	0.000	0	0	0.02128	0.02273	0.00871
[15,]	Hungary	18	589	428	[23,]	8.992	0	0	0.02941	0.03049	0.01574
[16,]	Ireland	18	965	140	[4,]	1.777	26	26	0.03571	0.04261	0.02527
[17,]	Italy	21	1,297	518	[13,]	5.599	26	27	0.03571	0.03935	0.02299
[18,]	Lithuania	23	3,496	2,906	[21,]	21.813	10	9	0.02083	0.00983	0.00146
[19,]	Luxembourg	18	1,199	25	[7,]	2.277	0	0	0.03030	0.03332	0.02208
[20,]	Latvia	14	491	1,674	[26,]	2.021	0	0	0.02273	0.01173	0.00179
[21,]	Malta	16	1,527	967	[24,]	0.964	0	0	0.02083	0.01322	0.00256
[22,]	Netherlands	15	234	71	[2,]	4.824	1	26	0.03333	0.04242	0.02532
[23,]	Poland	1	9	13	[19,]	0.000	0	0	0.03226	0.04193	0.02408
[24,]	Portugal	19	1,067	2,508	[22,]	3.320	0	0	0.03030	0.04142	0.02581
[25,]	Romania	21	4,567	1,396	[27,]	10.190	44	111	0.02083	0.01495	0.00550
[26,]	Slovakia	12	2,228	2,096	[20,]	0.189	0	0	0.02632	0.02022	0.00969
[27,]	Slovenia	21	3,153	73	[12,]	0.665	0	0	0.02381	0.01425	0.00539
[28,]	Sweden	11	315	68	[5,]	1.000	0	0	0.03571	0.03967	0.02322

Table 28 – Outcomes for each EU-28 country, remittance variable, 2011: Node Degree, Import Value (Out-Degree), HDI EU Rank and Betweenness and Closeness, considering different levels of alpha